Multi-Market Effects of Financial Transaction Taxes*

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Abstract

This paper investigates the effect of transaction taxes in a multi-market setting. We develop a model with adverse selection in which informed and liquidity traders may trade in equity and option markets. We find an asymmetric response to the same tax rate across markets due to the leverage of options. Stock relative to option volume decreases significantly and liquidity improves due to an alleviation of the adverse selection problem. When the effects of taxation on market-making competition are considered, the impact on market liquidity becomes ambiguous. We leverage quasi-random experiments in France, Italy and Spain to test our model’s hypotheses with high-frequency data on equity, derivative, and equity OTC markets. We show trading migration and a significant reduction in volume for all three countries, while only for Italy there is an effect on aggregate liquidity. These results can be linked to the different tax designs across countries, thus showing the importance of multi-market considerations for the optimal design of financial transaction taxes.

JEL-Code: D40, D53, D82, F38, G10, H26

Keywords: Financial transaction tax, liquidity, adverse selection, tax avoidance

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1 Introduction

The introduction of a European financial transaction tax (FTT) has been under discussion since 2011 when the EU Commission issued its first proposal. While a pan-European FTT failed to obtain unanimous support by all EU member states, the tax has been implemented unilaterally by France, Italy, and Spain. More recently, the former finance minister and prospective chancellor of Germany together with its French counterpart renewed the FTT proposal supported by the recent presidency of the EU council. With the FTT already in place in France and with the strong German support, a pan-European FTT is highly probable.

While this recent development is restricted to the EU, forms of FTTs have been proposed and introduced many times and in numerous countries. But regardless of its various implementations and the prolific literature it has sparked, the literature is silent about the FTT’s effects on multiple markets. What are the effects of an FTT across connected markets like stocks and their derivatives? When taxing stocks, does trading volume migrate to its derivative market through synthetic replication? How does a tax affect the composition of informed and liquidity traders? When taxing OTC trading activity, does it crop up on-exchange? Should the same tax rate be applied to the underlying and its derivative? We address these important questions and provide the first theoretical and empirical study of the impact of an FTT on multiple markets.

For the theoretical part of our analysis, the approach taken in this paper is based on the idea that an increase in trading costs through a tax in one market sets the incentive to migrate to another market for the same security or its derivative instrument. For example, taxing on-exchange financial activity encourages migration to OTC trading or synthetic replication of the stock with its equity derivative. In this spirit, we develop a multi-market sequential trade model that incorporates both stock and its option market. Our model provides an explicit characterization of trading incentives with the possibilities to trade in stock, its derivative, or both markets. The key element of our model is that we allow

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1Germany and France outlined a joint proposal for a financial transaction tax for the European Union that is based on the French model in December 2018. In December 2019, the Germany issues amended financial transaction tax proposal. In February 2021, the Portuguese presidency of the EU Council proposed an inclusive discussion among all EU Member States regarding tax design issues of the European FTT.

2As is well known, the first notable proponent of an FTT was Keynes (1936). Since then, important contribution have been made, such as e.g. Tobin (1978), Summers and Summers (1989), Stiglitz (1989), Krugman (2009). As of 2021, the following countries have some form of FTT in place: Belgium, China, Colombia, Finland, France, India, Italy, Spain, Peru, Poland, Singapore, Switzerland, Taiwan, United Kingdom, United States.

3The findings of an FTT can be without loss of generality transferred to an exogenous increase of trading costs (Dávila 2021).
informed traders to choose whether to buy or sell the stock, buy or sell a put, or buy or sell a call conditional on either or both markets being taxed. Market makers face adverse selection in the price-setting problem in both markets due to asymmetric information. We determine the conditions under which, in equilibrium, informed traders use options, stocks, or a mixture of the two assets to realize their profits. Additionally, we extend our analysis to include possible effects of taxation on liquidity provision. The model allows us to explicitly derive measures for trading activity and market liquidity. We contribute to the literature by proposing a two-market microstructure model that characterizes the migration of volume, the composition of market participants, and the consequent impact on market quality under asymmetric information and FTTs.

The model’s main findings are threefold. First, the model predicts an asymmetric effect of taxes levied on stocks compared to taxes on options. Due to the payoff structure and leverage of options, traders in our model reduce trading quantities less forcefully in the derivative market relative to the underlying’s market when confronted with a tax. This effect is pervasive throughout our model and also translates into a smaller effect of any tax regime on option markets compared to stock markets. Second, our model predicts a substantial reduction of trading volume in the stock market when a tax on stocks is introduced. This result is a combination of both a decrease in traded quantities as well as lower trading frequencies. Third, our model predicts increased liquidity in terms of lower spreads for the market affected by taxation. This result is due to alleviation of the adverse selection problem resulting from a shift of informed trading towards the untaxed market. Fourth, we also extend our model to investigate the possible effects of decreased competition among market makers due to increased costs of providing liquidity. When this effect is included, the effects on market liquidity are ambiguous. The model allows us to measure the relative effects of both mechanisms employing the relative impact of taxation on the revenues of liquidity providers and the cost of adverse selection.

For the empirical test of the theoretical model, we leverage quasi-random experiments to estimate the causal effects of FTTs on stock, its derivative, and equity OTC markets. Recent FTT introductions provide expedient events where either only stocks (France, Spain) or stock and its derivatives (Italy) are taxed. For all three cases, the tax includes equity OTC markets. It allows us to individually research the impact of FTT introductions on exclusively: (i) stocks on- and off-exchange with the same tax rate, (ii) stocks with a higher tax rate off-exchange than on-exchange and (iii) derivatives. We find that FTTs introduced only on stocks and with the same tax rate across on-exchange and OTC markets significantly affect trading activity, but not aggregate liquidity in stock markets. Market liquidity
or trading activity of on-exchange derivatives are not affected. We do not find evidence in the support of the common conception that the synthetic construction of stocks is a workaround of the tax. This is true for both stock options and futures. On the other hand, we do find spillover effects from OTC equity trading activity to on-exchange when different tax rates are implemented. In case (i), a 0.2% tax introduction decreases on-exchange Euro trading volume by 9 - 28%. In case (ii), tax introductions of 0.1% on-exchange and 0.2% off-exchange have no effect on-exchange and a decrease of 60% off-exchange. We explain the lack of response on-exchange with trade migration from the OTC markets. In case (iii), the tax rate on derivatives, which follows a fixed sliding scale, has no effect neither on trading activity nor market quality. The empirical results confirm three of the model’s predictions: (1) the effect of a tax on stocks relative to their derivatives is asymmetric, (2) stock volume decreases significantly while option volume is unaffected and (3) the effect on liquidity is ambiguous. Empirically, we do not find an improvement of market liquidity due to the alleviation of the adverse selection problem. The key contribution of the empirical part of the paper is therefore twofold. It lies in the individual effects as well as the comparison of the cross-country variation in policy design. Overall, our results highlight the importance of considering multi-market effects when designing optimal taxation that maximizes government revenues and minimize distortionary deadweight losses.

The remainder of this paper is organized as follows. Section 2 discusses related theoretical and empirical literature. Section 3 introduces the setting of our benchmark model. Section 4 derives the equilibrium conditions. Section 5 introduces taxation and extends the model to imperfect competition. Section 6 introduces the sequential model and shows the time-series effect of taxation. Section 7 and 8 describe the natural experiments exploited in the empirical applications and the data. Section 9 shows our empirical results and section 10 concludes.

2 Related Literature

To our knowledge, there exist no theoretical studies that examine the effect of transaction taxes across spot and its derivative markets. Various studies have investigated the effect of transaction taxes on market liquidity in a single-market setting in single markets. In general, these studies do not draw an unambiguous overall conclusion on the directional effect of a tax. Subrahmanyam [1998] finds that a tax can both increase or decrease market liquidity, depending on whether informed traders act competitively or in a monopolistic
way. Additionally, he finds that a tax can have positive effects by incentivising agents to acquire more long-term information than to short-term information. Similarly, Dupont and Lee (2007) find that the effect of a tax can be both negative or positive, depending on the level of informational asymmetry in the market. Dow and Rahi (2000) study the effect of a tax on the profits of speculators and the risk-sharing opportunities for hedgers. Again, they find that the effect of the tax depends on informational parameters of the model. These models have in common that they rely on a standard noise trading formulation. That is, the non-payoff related trading is modeled as an exogenous random shock to the market. As pointed out by Dávila and Parlatore (2020), this modeling approach is not appropriate to study the effects of trading costs, since in these models it is hard to understand how the behavior of noise traders varies with the level of trading costs. To avoid this problem, Dávila and Parlatore (2020) study the effect of trading costs on price informativeness using random heterogeneous priors. This formulation allows for trading motives that are not related to the payoff, therefore rendering prices only partially informative about the asset payoff while at the same time allowing taxes to affect the whole population of traders.

Empirical literature on the effects of FTTs are almost entirely focused on the taxation of stocks. The empirical findings for the FTT introduction in France come to a fairly clear conclusion - trading volume decreases while liquidity is essentially unaffected. Colliard and Hoffmann (2017) find no evidence that the FTT in France affects the composition of trading volume as proposed by Stiglitz (1989). They as well as Gomber, Haferkorn, and Zimmermann (2016) report a drop in trading volume by 10-20% and 15%, respectively. Both find no effect on liquidity on aggregate. Further, Colliard and Hoffmann (2017) show a shift from short- to long-term investors for the stocks treated with the tax. This is in line with the theoretical work by Amihud and Mendelson (1986) that implies that investors with longer holding periods own securities with higher transaction costs. Cappelletti, Guazzarotti, and Tommasino (2017) find no effect of trading volume for the FTT introduction in Italy but show with quarterly aggregated data for the entire Italian OTC market a mean difference of around 40% in trading volume. Coelho (2016) supports the findings for France as well as for Italy for the overall impact of the FTT introductions. She further shows that

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4See Matheson (2011) for a detailed discussion of other FTTs such as currency transaction tax (CTT) or Tobin tax, capital levy or registration tax and bank transaction tax (BTT). More recent, Feige (2001) proposes a tax on payments.

5See Colliard and Hoffmann (2017); Meyer, Wagener, and Weinhardt (2015); Becchetti, Ferrari, and Trenta (2014); Capelle-Blancard and Havrylych (2017); Coelho (2016); Gomber, Haferkorn, and Zimmermann (2016)

6Becchetti, Ferrari, and Trenta (2014) find a 30-37% drop in volume but they (i) do not account for strong seasonality and (ii) consider non-taxed stocks as the control group which are substantially affected by spillover effects which inflates the results significantly.
the strongest behavioral response comes from high-frequency trading. The empirical work relating transaction taxes to derivatives markets is almost nonexistent. To our knowledge, there is only the work by Mixon (2021) on the US transaction tax on futures during the 1920s and 1930s. It shows a decline in trading volume due to reduced intra-day trading and that intermediaries doubled the minimum tick size to pass through the tax to customers.

This paper is also related to various theoretical studies that investigate different aspects of the introduction of derivatives to its underlying assets. Brennan and Cao (1996) and Huang (2016) extend the canonical framework developed in Hellwig (1980) and Grossman (1980) to include a derivative market. Brennan and Cao (1996) show that introducing an option type contract allows to achieve Pareto efficiency with one round of trading, which, in turn, leads traders to stop trading in both the stock and option after the first trading period. Furthermore, they find that market depth, as defined in Kyle (1985), increases with the introduction of a derivative. Huang (2016), on the other hand, look at the effect of an option market on information acquisition. Compared to Brennan and Cao (1996), they introduce a set of option contracts with different strike prices. They find that the introduction of a continuous set of options increases the incentive of acquiring private information if the information acquisition cost is high. This in turn increases the price informativeness and asset prices while it decreases price volatility and response to earnings announcements. The opposite is found when information acquisition costs are low. Both Brennan and Cao (1996) and Huang (2016) use the classic noise trading formulations in their models. The same is true for Easley, Hara, and Srinivas (1998), but differently to Huang (2016) and Brennan and Cao (1996), they develop a model with a binomial payoff structure and risk neutral, competitive market makers. They are able to establish conditions under which informed traders trade options, and consequently, convey information for the stock market. Cao and Ou Yang (2009) develop a framework similar to Brennan and Cao (1996), where agents trade due to differences in opinion rather than asymmetric information. They also show that a Pareto optimal allocation can be achieved with the introduction of an option market. Importantly though, they are able to endogenously generate trading beyond the first trading round in a dynamic model by differential interpretation of public signals, whereas in Brennan and Cao (1996) additional noise trading needs to be assumed for trading volume to be positive after the first trading period. Gao and Wang (2017) investigate the introduction of an option market in a setting similar to Vayanos and Wang (2012), where agents have heterogeneous uncertain endowment and information. In this setting, an option market does not increase the informational efficiency of the market, but, as in Brennan and Cao (1996) and Cao and Ou Yang (2009), it increases the allocational
efficiency. Furthermore, in their model the reason for agents trading in options is the disagreement about the payoffs uncertainty, which is similar to the findings in Cao and Ou Yang (2009).

### 3 Stylized Model

We consider a one-period model with a risk-free asset, one stock, and a set of options on the asset. The return on the risk-free asset is normalized to zero. The market is populated by two different types of traders and a risk-neutral market makers that set the prices for the different assets after observing the order flow. The orders are placed by either a strategic, perfectly informed trader (IT, insider) or a competitive, rational, and risk-averse liquidity trader (LT). Furthermore, there exist three states of the world, which determine the final value of the stock at the end of the period. The insiders know the final state and trade to maximize their profits. The liquidity traders are exposed to state-contingent shocks and trade to hedge their risk exposure.

#### 3.1 Market Structure

At the beginning of each trading day, an information event happens with probability $\eta$. Conditional on an information event happening, the final value of the asset is either $S_0 u$ or $S_0 d$ with probabilities $\mu$ and $(1 - \mu)$ respectively, where $S_0$ is the value of the stock at the beginning of the trading day. If no information event happens, the final value of the asset is $S_0 m$, with $u \geq m \geq d$. We will refer to the three states of the world as the "u-state" (up-state), "d-state" (down-state) and the "m-state" (middle-state). For the derivative contracts, we restrict our attention to two options, a (European) call or put option with expiration at the end of the period and strike prices $K_c$ and $K_p$ respectively. The orders placed by the traders are market orders. Market makers for both the risky asset and the options set their prices after observing an order that stems either from an insider or from a liquidity trader. Furthermore, it is assumed that the market makers in both the option and stock market have access to the same information set.

#### 3.2 Traders

The market is populated by a unit measure of traders, a fraction $\delta$ of which are informed insiders, and the remaining fraction $(1 - \delta)$ are liquidity traders. Insiders perfectly anticipate
the final state and therefore the stock and option payoff. Additionally, they are risk-neutral and strategic, trade to maximize their profits, and do not face any borrowing or short-selling constraints.

The liquidity traders, on the other hand, are competitive and accordingly do not anticipate the impact of their orders on market prices. They choose their trades to maximize the utility of their terminal wealth, with utility function given by $U(W) = W - \gamma W^2$, where $\gamma$ determines the risk aversion of the liquidity traders. The liquidity traders have state-contingent shocks and therefore trade in the stock and option market to hedge their risk exposure. Liquidity traders can experience two different type of shocks. $LT_1$ is exposed to a liquidity shock equal to $-L$ in the $u$-state, $-l$ in the $m$-state and 0 in the $d$-state, whereas $LT_2$ is exposed to state-contingent shocks equal to 0, $-l$ and $-L$ for states $u$, $m$ and $d$ respectively. Finally, liquidity traders will trade in the stock market with probability $\omega$ or in the option market with probability $(1 - \omega)$. We therefore have four different type of liquidity traders, depending on the type of shock they experience and the market they use to hedge their risk exposure ($\{LT_1^S, LT_2^S, LT_1^O, LT_2^O\}$).

Contrary to liquidity traders, informed traders can choose in which market they wish to trade. Therefore, a trader who is informed that the world is currently in an up-state will choose to trade either in the stock or option market based on the profits that he is able to make in either market. Naturally, in order to profit from the up-state, an informed trader will either buy a stock, buy a call or sell a put. Similarly, a trader informed of a down-state will either sell the stock, sell a call or buy a put. The profits available to an informed trader if $S = S_0u$ are therefore

$$\Pi_{up} = \begin{cases} 
Q_s(S_0u - A_s) & \text{if buy stock} \\
Q_c((S_0u - K_c)\theta - A_c) & \text{if buy call} \\
Q_pB_p & \text{if sell put.}
\end{cases} \quad (1)$$

and similarly for the down-state.

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For simplicity we further assume that liquidity traders, conditional on trading in the option market, will trade either calls or puts. This is akin to allowing for a unit measure of liquidity traders that trade either in calls or puts with defined probabilities. In general, the assumption that liquidity traders in our model trade either the stock, a call or a put option is innocuous, in that it does not affect the main results of our model.
\[ \Pi_{\text{down}} = \begin{cases} 
Q_s(B_s - S_0d) & \text{if sell stock} \\
Q_cB_c & \text{if sell call} \\
Q_p((K_p - S_0d)\theta - A_p) & \text{if buy put,} 
\end{cases} \]

where \( \{A_s, B_s, A_c, B_c, A_p, B_p\} \) are respectively the bid and ask prices for stock, call and put, \( \{Q^*_s, Q^*_c, Q^*_p\} \) are the trading strategies available to the informed trader and \( \theta \) is the number of stocks controlled by an option contract. An informed trader will therefore decide in which market to trade based on these profits. The probabilities that an informed trader trades in the stock market conditional on the state, which we denote by \( \{\alpha_u, \alpha_d\} \) as well as the probabilities that informed traders choose calls if they trade in the option market \( \{\beta_u, \beta_d\} \) will therefore be determined in equilibrium.

Before outlining the equilibrium, it is worth briefly summarizing the game-theoretical nature of the model. Figure 1 shows the trading structure and main parameters of the game considered in this model. We can see that the first two moves in the game are nature’s decisions regarding the existence and type of information that is going to affect the markets for the stock and options. These events can be thought of as events that happen prior to the opening of the markets (or, similarly, information events that happen overnight if multiple trading days are considered). Given the known probabilities \( \{\eta, \mu, \delta, \omega\} \), the market makers will set their initial prices in their respective markets. Finally, a trader is randomly chosen from the population of traders and a trade outcome occurs.

### 3.3 Equilibrium Notion

The equilibrium in the economy above is a standard rational expectation equilibrium, characterized by a set of prices and trading strategies, such that:

1. The liquidity traders choose market orders to maximize their expected utility
2. The insiders choose their market orders to maximize their profits
3. The market makers set prices equal to their conditional expectations
Figure 1: The tree diagram summarizes the probabilistic structure of the model described in this section. At the LTS nodes the game develops as in the case where no information event happens. $LT^S$, $LT^C$ and $LT^P$ refer to the liquidity traders that trade stocks, call options or put options respectively. $LT_1$ and $LT_2$ refer to the type of liquidity shocks experienced by the liquidity traders.
4 Equilibrium Prices and Trades

4.1 Liquidity Traders

As mentioned above, the liquidity traders trade to hedge their liquidity shocks. Since $LT_1^S$ receives negative shocks in the $u$ and $m$ state, it is reasonable to assume that she will therefore buy the stock. In contrast, $LT_2^S$ will sell the stock to hedge against the liquidity shocks she is exposed to. Similarly, the liquidity traders who trade in the option market will either buy a call, sell a put, sell a call or buy a put.

The objective functions for the liquidity traders who trade in the stock market are therefore:

$$Q_s^{1*} = \arg \max_{\{Q_s\}} \mathbb{E}[U(Q_s)] = \max_{\{Q_s\}} \left[ \eta \mu U[-L + (S_0 u - A_s)Q_s] \right.$$ 
$$+ (1 - \eta)U[-l + (S_0 m - A_s)Q_s] + \eta(1 - \mu)U[(S_0 d - A_s)Q_s] \right]$$ 

$$+ \eta(1 - \mu)U[(S_0 d - A_s)Q_s]$$

$$Q_s^{2*} = \arg \max_{\{Q_s\}} \mathbb{E}[U(Q_s)] = \max_{\{Q_s\}} \left[ \eta \mu U[-L + (S_0 u - A_s)Q_s] \right.$$ 
$$+ (1 - \eta)U[-l + (S_0 m - A_s)Q_s] + \eta(1 - \mu)U[(S_0 d - A_s)Q_s]$$

For the liquidity traders who trade in the option market the objective functions are:

$$Q_c^{1*} = \arg \max_{\{Q_c\}} \mathbb{E}[U(Q_c)] = \max_{\{Q_c\}} \left[ \eta \mu U[-L + ((S_0 u - K_c)^+ \theta - A_c)Q_c] \right.$$ 
$$+ (1 - \eta)U[-l + ((S_0 m - K_c)^+ \theta - A_c)Q_c] + \eta(1 - \mu)U[((S_0 d - K_c)^+ \theta - A_c)Q_c]$$

$$+ \eta(1 - \mu)U[((S_0 d - K_c)^+ \theta - A_c)Q_c]$$
\[ Q^*_p = \arg \max_{\{Q_p\}} \mathbb{E}[U(Q_c)] = \max_{\{Q_s\}} \left[ \eta \mu U[-L(B_p - (K_p - S_0 u)^+ \theta)Q_p] + (1 - \eta)U[-l + (B_p - (K_p - S_0 m)^+ \theta)Q_p] + \eta(1 - \mu)U[(B_p - (K_p - S_0 d)^+ \theta)Q_p] \right]. \] (6)

and similarly for \( LT^C_2 \) and \( LT^P_2 \).

**Lemma 1** The liquidity traders who trade in the stock market buy (sell) \( Q^*_s \) (\( Q^*_{s_2} \)) shares of stock and the liquidity traders who trade in the option market buy (sell) \( Q^*_c \) (\( Q^{2*}_c \)) call option contracts and \( Q^*_p \) (\( Q^{2*}_p \)) put option contracts, where

\[ Q^*_s = \text{Max}[0, f_{QS1}] \quad \text{and} \quad Q^{2*}_s = \text{Min}[0, f_{QS2}], \] (7)

\[ Q^*_c = \text{Max}[0, f_{QC1}] \quad \text{and} \quad Q^{2*}_c = \text{Min}[0, f_{QC2}], \] (8)

\[ Q^*_p = \text{Max}[0, f_{QP1}] \quad \text{and} \quad Q^{2*}_p = \text{Min}[0, f_{QP2}], \] (9)

where the functions \( f_{QS1}(\cdot), f_{QS2}(\cdot), f_{QC1}(\cdot), f_{QC2}(\cdot), f_{QP1}(\cdot) \) and \( f_{QP2}(\cdot) \) are defined in the appendix.

### 4.2 Insiders

The insiders in this model will always mimic one of the trades followed by the liquidity traders. The intuition is the following. Let us assume that we are in state \( u \). Since the insiders are perfectly informed, they will rather choose to buy the stock, buy the call option or sell the put option rather then selling the stock, buying the put or selling a call option. Given that they will buy the stock, buy the call or sell the put, they could choose to trade another amount of stocks and calls compared to the liquidity trader. Since the market maker has rational expectations and perfectly anticipates the strategies of the participants in the market, she will infer that any other amount than \( Q^*_s, Q^*_c \) or \( Q^*_p \) will necessarily
come from the insider. In this case, she will set the price equal to the expected value, which in this case is equal to $S_0u$, making the trades unprofitable for the insider.

### 4.3 Prices

As mentioned above, the market makers set the price equal to the expected value conditional on her information. Therefore, the ask and bid price for the first trade of day 1 are given by:

$$A_s = \mathbb{E}[S \mid Qs > 0] = S_0uPr[u \mid Qs > 0] + S_0mPr[m \mid Qs > 0] + S_0dPr[d \mid Qs > 0]$$

$$= \frac{S_0((m - m\eta + d(-1 + \delta)\eta(1 + \mu))\omega + u\eta\mu(2\alpha_u\delta + \omega - \delta\omega))}{2\alpha_u\delta\eta\mu + \omega - \delta\omega}$$

$$B_s = \mathbb{E}[S \mid Qs < 0] = S_0uPr[u \mid Qs < 0] + S_0mPr[m \mid Qs < 0] + S_0dPr[d \mid Qs < 0]$$

$$= \frac{S_0(m(-1 + \eta)\omega + u(-1 + \delta)\eta\mu\omega + d\eta(-1 + \mu)(2\alpha_d\delta + \omega - \delta\omega))}{2\alpha_d\delta\eta(-1 + \mu) + (-1 + \delta\eta)\omega}$$

Similarly, the bid price of the call option is given by

$$B_c = \mathbb{E}[\text{Call} \mid Qc > 0]$$

$$= (K_p - S_0u)^+\theta Pr[u \mid Qp < 0] + (K_p - S_0m)^+\theta Pr[m \mid Qp < 0]$$

$$+ (K_p - S_0d)^+\theta Pr[d \mid Qp < 0]$$

$$= \frac{\theta(K_c(1 - \eta - (\delta - 1)\eta\mu) - S_0(m - m\eta - u(-1 + \delta)\eta\mu))(1 + \delta)}{1 - \omega + \delta\eta(-1 + 4(-1 + \alpha_d)\beta_d(-1 + \mu) + \omega)}.$$ 

The remaining prices can be computed in the same way. Note that these quantities depend on the probabilities that informed traders either trade in the stock or the option market. Next we will therefore determine the conditions for when informed traders will trade in each market and the equilibrium probabilities of informed trading.

For the remainder of this section we assume $\eta = 1$, $m = 1$, $K_c = S_d$, $K_p = S_u$ and $S_0 = 1$ in order to facilitate the exposition of the results.
4.4 Informed Trading

Intuitively, there exist two types of equilibria: one where informed traders only choose to trade in either the stock market or the option market (e.g. \( \alpha_u, \alpha_d \) are at their boundaries) and one where informed traders are indifferent between trading in the two markets (e.g. \( \alpha_u, \alpha_d \) are inside their boundaries). We will call the former a separating equilibrium and the latter a pooling equilibrium. The conditions for a pooling equilibrium are such that, conditional on all insiders trading in the stock market, the profits of trading in the option market are higher than the profits of trading in the stock market. We provide calculations for the case where the trader is informed that we are currently in an up-state. The conditions for a pooling equilibrium in the down-state are then found in a similar fashion.

We know that an informed trader in the up-state will either buy the stock, buy the call or sell the put. The profit of buying a stock, given that \( \alpha_u = 1 \), is

\[
\Pi(BS) = \frac{(1-\delta)(1-\mu)\omega(\delta - L\gamma\omega + L\gamma\delta\omega)}{\gamma(2\delta\omega(\omega - 2\mu) - \omega^2 + \delta^2(4\mu(\omega - 1) - \omega^2))}, \tag{13}
\]

and the expected profit from buying the call or selling the put is

\[
\Pi(BC) = \Pi(SP) = L(1 - \mu). \tag{14}
\]

The conditions for a pooling equilibrium in the up-state are therefore

\[
L(1 - \mu) > \frac{(1-\delta)(1-\mu)\omega(\delta - L\gamma\omega + L\gamma\delta\omega)}{\gamma(2\delta\omega(\omega - 2\mu) - \omega^2 + \delta^2(4\mu(\omega - 1) - \omega^2))}. \tag{15}
\]

Whenever this condition holds, at least some of the informed traders will choose to trade in the option market. In that case we will be in a pooling equilibrium, as liquidity and informed traders will ”pool” together in the option market. Alternatively, whenever this condition does not hold, all informed traders will choose to trade in the option market, as that will maximize their profits. That would therefore be a separating equilibrium. The proposition below summarizes the equilibrium of the static benchmark model.

**Proposition 1** For the market structure described above we have a pooling equilibrium if and only if
\[\frac{\delta(1 - \mu)(4L\gamma\mu(\delta(-1 + \omega) - \omega) + (-1 + \delta)\omega)}{\gamma(-\omega^2 + 2\delta + \omega(-2\mu + \omega) + \delta^2(4\mu(-1 + \omega) - \omega^2))} > 0 \quad (16)\]

and

\[\frac{\mu((1 - \delta)\delta\omega - \omega(-1 + \gamma(-1 + \delta)^2\omega^2))}{\gamma(\delta^2((-2 + \omega)^2 + 4\mu(-1 + \omega)) + \omega^2 - 2\delta\omega(-2 + 2\mu + \omega))} > 0, \quad (17)\]

and the equilibrium \{\alpha_u, \alpha_d\} and \{beta_u, \beta_d\} are then given by

\[\alpha_u^* = \alpha_d^* = \omega \quad \text{and} \quad \beta_u^* = \beta_d^* = \frac{1}{2} \quad (18)\]

The equilibrium trading strategies are

\[Q_s^1 = \frac{L\gamma(-1 + \delta)(1 + \delta(-1 + 2\mu))}{(d - u)\gamma(1 + \delta^2 + \delta(-2 + 4\mu))}, \quad (19)\]

\[Q_c^1 = Q_p^1 = \frac{L\gamma(-1 + \delta)(1 + \delta(-1 + 2\mu))}{(d - u)\gamma\theta(1 + \delta^2 + \delta(-2 + 4\mu))}, \quad (20)\]

\[Q_s^2 = \frac{L\gamma(-1 + \delta)(-1 + \delta(-1 + 2\mu))}{(u - d)\gamma(1 + \delta^2 + \delta(2 + 4\mu))}, \quad (21)\]

\[Q_c^2 = Q_p^2 = \frac{L\gamma(-1 + \delta)(1 + \delta(-1 + 2\mu))}{(u - d)\gamma\theta(1 + \delta^2 + \delta(2 + 4\mu))}, \quad (22)\]

and the equilibrium prices are

\[A_s = \frac{d(-1 + \delta)(-1 + \mu) + u(1 + \delta)\mu}{1 + \delta(-1 + 2\mu)} \quad B_s = \frac{\delta(1 + \delta)(\mu - 1) + u(\delta - 1)\mu}{\delta(2\mu - 1) - 1} \quad (23)\]
\[
A_c = \frac{(u - d)(1 + \delta)\theta\mu}{1 + \delta(2\mu - 1)} \quad B_c = \frac{(u - d)(\delta - 1)\theta\mu}{\delta(2\mu - 1) - 1}
\] (24)

\[
A_p = \frac{(u - d)(1 + \delta)\theta(\mu - 1)}{\delta(2\mu - 1) - 1} \quad B_p = \frac{(u - d)(\delta - 1)\theta(\mu - 1)}{1 + \delta(2\mu - 1)}
\] (25)

We note that the conditions for a pooling equilibrium are generally satisfied when stock market liquidity \((\omega)\) is low, when the the risk aversion \((\gamma)\) of the liquidity traders is high, when the liquidity shock \((L, l)\) are high and when the proportion of informed trading \((\delta)\) is low\(^8\). Furthermore, the equilibrium probabilities of informed trading in the stock market turns out to be equal to the liquidity in these markets. This result, even though surprising at first glance, is rather intuitive: When there is a high liquidity in the stock market, informed trading will be harder to discover for the market maker. This means that informed trades will have a smaller impact on stock prices, which in turn allows them to extract higher profits from these markets. Lastly, the equilibrium quantities for stock and option trades only differ through the leverage effect of the options.

### 4.5 Comparative Statics

Next, we show how model equilibria are affected by model parameters. For this purpose we drop the main simplifying assumption made previously, e.g. \(\eta = 1\).\(^9\) Figure 2 shows how the pooling equilibrium conditions in the up-state are affected by the main parameters of the model.\(^10\) We see that increasing \(\mu, \eta, \) and \(\delta\) increases the likelihood of a pooling equilibrium. Intuitively, increasing the probability of an up-state \((\mu)\) will increase the probabilities that ”up-state trades” (e.g. buying the stock, buying a call or selling a put) will occur. Given that these conditions are derived for \(\alpha_u = 1\), this will disproportionally increase the probability that a buy order in the stock market comes from an informed trader, making it necessary for the market makers to increase stock prices accordingly.

\(^8\)Note that, in our stylized model, the simplifying assumptions make sure that the condition for a pooling equilibrium always hold. This is not the case if the assumptions are relaxed or taxation is introduced in the model.

\(^9\)For the remainder this section, unless stated otherwise, the baseline parameters used for the comparative statics are \(\eta = 0.5, \mu = 0.5, \omega = 0.5, \delta = 0.2, \theta = 5, \gamma = 2, L = 0.9, l = 0.2, u = 1.2, d = 0.8, m = 1, K_c = d, K_p = u\).

\(^10\)We only consider the up-state since the results for the down-state are symmetrical.
This will in turn make trades in the option market more profitable, making it more likely that a pooling equilibrium exists.

Figure 2: The Figure shows the effect of the probability of an up-state ($\mu$), the proportion of liquidity traders in the stock market ($\omega$), probability of an information event ($\eta$) and the proportion of informed trading ($\delta$) on the conditions of a pooling equilibrium, e.g. $\Pi(BC) - \Pi(BS)$ and $\Pi(SP) - \Pi(BS)$.

The intuition is the same for an increase in the probability that an information event happens and the proportion of informed trading in the market. In both cases, the underlying mechanism that leads to an increased likelihood of a pooling equilibrium is that the relative presence of informed trading in the stock market increases, leading to higher prices in the stock market and accordingly to smaller profit margins for informed traders that trade stocks. The opposite is true for an increase in $\omega$. By increasing the proportion of liquidity traders in the stock market, naturally the relative presence of informed trading will be smaller, making it easier for informed traders to extract profits by trading stocks.

Figure 3 shows the equilibrium ask and bid prices as a function of $\delta$, $\eta$, $\mu$ and $\theta$. For $\delta$, $\eta$, the mechanism described above, e.g. an increase in the relative presence of informed trading, naturally also leads to higher spreads in the affected markets. The effect of the leverage of options ($\theta$) has the straightforward effect of increasing the level as well as the
spread of option prices, while not affecting the stock prices.

Figure 3: The Figure shows the effect of $\delta$, $\eta$, $\mu$ and $\theta$ on the equilibrium Ask and Bid Prices.

The effect of $\mu$ on the equilibrium prices on the other hand is more nuanced. On the one hand, increasing the likelihood of an up-state (down-state) naturally increases (decreases) the level of the prices. In addition to this effect, Figure 3 also shows that the spreads in both market slightly increase as $\mu$ moves away from its boundaries. This effect arises due to the higher uncertainty in the market when $\mu = 0.5$, as in this case up and down movements in the markets are equally likely. Finally, it is interesting to note that changing $\omega$ does not have any effect on the equilibrium prices. This result could seem counter-intuitive at first glance, but it actually arises due to the equilibrium (Equation 19) shown in Proposition 1. In fact, the decrease in spreads in the stock market that one might expect from an increase in $\omega$ is countered by an increase in informed trading. In equilibrium, these effects essentially cancel each other out. As we will see in the next sections, this result no longer holds once transaction taxes are introduced.
5 Transaction Taxes

Next, we introduce transaction taxes in the markets and analyse how they affects the interaction between the two markets. We consider three different scenarios, one where the tax is only introduced in the stock market, one where it is only introduced in the option market, and one where a tax exists in both markets. The transaction taxes are introduced in the following fashion.

**Stock Market**

The tax is introduced as a percentage cost on the value of the transaction. Therefore, when buying (selling) a stock, the agent in the market will pay (receive) \( A_s(1 + t) \) or \( (1 - t)) \), where \( t \) is the transaction tax.

**Option Market**

Next, we look at how our dual market system is affected by an introduction of a tax in the option market. Again, the tax is introduced as a percentage cost of the value of the transaction, therefore, when trading in the option market the agents will pay (receive) \( A_c(1 + t) \) or \( A_p(1 + t) \) and \( B_c(1 - t) \) or \( B_p(1 - t) \).

**Both Markets**

Finally, in the last scenario a tax is introduced in both markets. Again, the tax is introduced as a percentage cost on the value of the transaction and the traders will therefore pay (receive) \( A_s(1 + t) \) or \( (1 - t)) \) when they trade in the stock market as well as pay (receive) \( A_c(1 + t) \) or \( A_p(1 + t) \) and \( B_c(1 - t) \) or \( B_p(1 - t) \) if they buy call options (put options).

The objective functions for the liquidity traders given the presence of transaction taxes change in a straightforward way, and the equilibrium results for the three scenarios described above are shown in the next section.

5.1 Equilibrium Investment with Taxes

First, we explore how taxes impact the conditions for a pooling equilibrium.

Figure 4 shows the difference between the profits available to the informed trader in the option and stock market as a function of some of the models parameters.\(^{11}\) As we would

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\(^{11}\)For the remainder of this section we will focus on the case of an up-state. The results for the down-state are symmetrical.
Figure 4: The Figure shows the effect of $\mu$, $\omega$, $\eta$ and $\delta$ on the conditions of a pooling equilibrium, e.g. $\Pi(BC) - \Pi(BS)$ and $\Pi(SP) - \Pi(BS)$

expect, with option taxes in the market it becomes less likely that the conditions for a pooling equilibrium are satisfied, as the profits in the option market are adversely affected by the taxes. Additionally, the difference in profits in the two markets is also slightly lower when taxes are introduced in both markets compared to the scenario when there exists only a tax in the stock market. The directional impact of the parameters on the conditions is the same as shown in Figure 2.

Next we look at the equilibrium probabilities of informed trading in either market. Figure 5 showcases an important result of our model. If the same tax function and rate is introduced for the stock and derivative market, the effect of taxation in the stock market will be larger than the effect in the derivative market. Figure 5 displays this result, which is pervasive throughout our remaining theoretical findings, as an asymmetric deviation of $\alpha_u(Option Tax)$ and $\alpha_u(Stock Tax)$ with respect to the benchmark case of no taxation. This result arises as a consequence of the different payoff structure of stocks compared to derivative products. Intuitively, due to the inherent leverage of derivative products, the price constitutes a smaller proportion of the final payoff compared to stocks. If a standard
tax on the value of the transaction is considered, as described in section 5, and the same tax rate is used for both markets, this will lead to a higher effect of taxation in stock markets compared to derivative markets. We provide further intuition as well as policy implications of this result in the Appendix and section 9.2 respectively.

In addition, Figure 5 shows that $\alpha^*_u$ and $\beta^*_u$ are not constant once taxes are considered. The effect of $\mu$, $\delta$ and $\eta$ on $\alpha^*_u$ is the same. For the case of an option tax, as these parameters increase, $\alpha^*_u$ decreases. This is due to the same logic discussed when describing Figure 2, namely that, as $\mu$ increases the probability of observing a buy in the stock market increases, leading the market maker to increase prices and informed traders to move to the option market. This holds only in the scenario when only an option tax is present, as in that case the presence of informed traders in the stock market will be naturally higher (due to the asymmetric effect described above) and there are enough profits available in the option market such that informed traders wish to move to the derivatives market. For the other two scenarios, $\alpha^*_u$ will be lower, as the stock market is taxed and returns are therefore decreased. Additionally, as $\mu$ is low, expected profits in the option market will be higher as there is a limited downside risk. As $\mu$ increases, stocks become more valuable relative to options, leading to a slight increase in $\alpha^*_u$. As $\delta$ and $\eta$ increase, the relative presence of informed trading in the market increases. As we can see, this leads to a fast increase of $\alpha^*_u$ for low values of the parameters. Naturally, as the presence of informed trading increases, the markets will naturally find an "equilibrium" amount of informed trading. This is clearly shown in the top right graph of Figure 5, where the level of informed trading remains almost constant for $\delta > 0.5$. Depending on the tax regime, this "equilibrium" level will be either above or below the equilibrium $\alpha^*_u$ found in the case of no taxation.

The effect of $\omega$ on $\alpha^*_u$ is straightforward. As the liquidity in the stock market increases, the proportion of informed trading in the stock market increases as well, no matter the tax regime. Finally, we look at the equilibrium quantities and prices for the three different tax regimes. Figure 6 shows that in every tax regime the equilibrium quantities traded are lower compared to the case without a tax. This result follows intuitively from the results shown in Figure 5 combined with the effect of taxation on the returns available in the market. For the case of a tax in the option market, we know from Figure 5 that the equilibrium informed trading will be slightly higher than the equilibrium proportion without taxation. This effect alone, without considering the effect of taxation on the return, would lead to higher quantities traded in the option market, resulting from improved liquidity conditions in the option market and simultaneous worse liquidity in the stock market. This effect is countered by decreased returns, which leads to slightly lower quantities traded in the case
Figure 5: The Figure shows the effect of $\mu$, $\omega$, $\eta$ and $\delta$ on the equilibrium probabilities of informed trading in the stock and option market. The red dashed line shows the equilibrium probabilities of informed trading for the benchmark case without taxation.

... of an option tax compared to a tax-less scenario, as shown in Figure 14. The same logic applies to the other two tax regimes.

Furthermore, the parameters considered in Figure 6 have different effects on the equilibrium quantities. As shown in the top left quadrant, the latter are an increasing function of $\mu$. As the probability of an up-state increases, these trades become more desirable, leading to increased quantities. Interestingly, the equilibrium quantities also increase as the liquidity shock $L$ increases. This is intuitive, as liquidity traders’ optimal way to hedge against liquidity shocks in the up state is to increase the exposure to trades that have positive payoffs in the up state. Increasing the proportion of informed trading in the market leads to an overall decrease in trading. For a certain threshold of informed trading the spreads in the markets will become too large, leading to the well-known result of a market breakdown in the spirit of Glosten and Milgrom (1985). Increasing the leverage of options $\theta$, as expected, does not affect equilibrium quantities in the stock market. Interestingly, increasing the amount of shares controlled by an option contract leads to lower quantities...
traded in the option market. This is due the adverse effect on option prices of an increased θ, which counters the effect of increasing the intrinsic value of the option. Figure 7 shows the equilibrium prices for the three tax regimes. In the top left graph we see that increasing the probability of an up-state increases the level of the prices, since the stock becomes more valuable the more likely it becomes that its value will increase. Additionally, we can see that the spreads slightly increase as μ → 0.5, since the uncertainty in the market is the highest when μ = 0.5. Increasing δ and η has the same effect of increasing the spreads in the market, as in both cases the relative presence of informed traders in the stock market increases, which in turn has an adverse effect on the spreads. The effect of increasing the presence of liquidity trading in the stock market has different effects depending on the tax regime. As can be noted from the bottom right graph in Figure 7 for a stock tax and a dual tax regime, increasing the liquidity in the stock market actually increases the spreads. This is due the fact that increasing ω simultaneously increases α_u*, where the ladder effect dominates the former one. The opposite is true in the case of an option tax, which leads
Figure 7: The Figure shows the equilibrium ask prices for the stock as functions of $\mu$, $\delta$, $\eta$ and $\omega$. The dashed line shows the equilibrium ask prices in the stock market w for the benchmark case without taxation.

to a slight decrease in spreads.

5.2 Imperfect Competition

Finally, to explore the effects of taxation on liquidity provision, we include an additional friction, namely imperfect competition (IC) among market makers. In the benchmark model considered so far, we considered a model where market makers, when chosen to trade are allotted the full order flow. As is well-known, this mechanism leads to market makers slightly undercutting each others quotes until prices eventually reach their conditional expectations and profits are driven down to zero. In this section we consider a different trading mechanism, namely a call auction. Dealers are now assumed to submit a quote schedule, that is, a set of quantities they are willing to trade for any given price. At every trading round, an auctioneer will then parcel out the full order flow among all the market makers at the market clearing price. More formally, we consider a fixed number of market makers $M$, a proportion $m_s$ of which provide liquidity in the stock market and
(1 − m_s) provide liquidity in the option market\textsuperscript{12}. We then define Q^m_s(p_s) and Q^m_o(p_o) to be the total number of shares or options that a dealer is willing to sell (or offer to buy if Q < 0) at price p. The total market supply for the two markets at price p is then \( \sum^M Q^m_s(p_s)m_s \) and \( \sum^M Q^m_o(p_o)(1 − m_s) \).

Recall that in the model of Section 3, the market makers have rational expectations about the proportion of trades arriving from different traders and their trading strategies, and set prices accordingly. With this trading mechanism, market makers also need to take into consideration the behavior of the other market makers and accordingly how their own strategies impact equilibrium prices. Therefore, in this section we seek a rational expectation equilibrium such that the set of schedules set by market makers \( \{Q^m(p)\}_{m=1}^M \) and the according price mapping \( p^*(q) \) is such that (i) it maximizes his profits, (ii) the market makers correctly anticipates the clearing price and forms expectations accordingly, (iii) the markets clear, e.g. \( \sum^M m_s Q^m_s(p_s) = Q_s \) and \( \sum^M (1 − m_s)Q^m_o(p_o) = Q_o \). The following proposition summarizes the equilibrium:

**Proposition 2** For the market mechanism described above, we have that market makers in the stock market will post the following quantity and prices schedules:

\[
Q^m_s(p) = \begin{cases} 
\phi_s p & \text{if } p = A_s \\
\frac{1}{\phi_s} p & \text{if } p = B_s 
\end{cases}
\text{ with } \phi_s = \frac{(M - 2)}{(M - 1)(Mm_s - m_s - M)\alpha_s}, \tag{26}
\]

\[
p_s(Q_s) = \begin{cases} 
\lambda_s p & \text{if } Q_s > 0 \\
\frac{1}{\lambda_s}Q_o & \text{if } Q_s < 0 
\end{cases}
\text{ with } \lambda_s = \frac{(M - 1)(1 - m_s + Mm_s)\alpha_s}{M(M - 2)m_s}, \tag{27}
\]

and the market makers in the option market will post the following schedules:

\[
Q^m_o(p) = \begin{cases} 
\phi_o p & \text{if } p = A_o \\
\frac{1}{\phi_o} p & \text{if } p = B_o 
\end{cases}
\text{ with } \phi_o = \frac{(M - 2)}{(M - 1)(M(1 - m_s) + m_s)\alpha_o}, \tag{28}
\]

\textsuperscript{12}We decide to not endogenize the choice of } m_s \text{, as it does not change the intuitions derived from this section. In order to make } m_s \text{ endogenous, we could use a similar mechanisms used for informed trading, that is, previous to every trading round, market makers would choose the optimal } m_s \text{ by equalizing the profits available in both the stock and the option markets.}
\[ p_o(Q_o) = \begin{cases} \lambda_o p & \text{if } Q_o > 0 \\ \frac{1}{\lambda_o} Q_o & \text{if } Q_o < 0 \end{cases} \quad \text{with} \quad \lambda_o = \frac{(M - 1)(M(1 - m_s) + m_s)\alpha_o}{M(M - 2)(1 - m_s)}. \quad (29) \]

Figure 8 visualizes the proposition above in terms of equilibrium prices. Specifically, Figure 8 shows how imperfect competition affects the equilibrium ask and bid prices - and therefore the spread - in the stock market as a function of \( \mu, \delta, \eta \) and \( \omega \). As is clear from Proposition 2, imperfect competition generally has an adverse effect on the liquidity of the markets. In our case, because we allow market makers to move from the stock to the option market, the liquidity (in terms of spreads) of the stock market will decrease (increase) as \( m_s \) decreases (increases).

![Figure 8](image_url)

Figure 8: The Figure shows the equilibrium stock prices as functions of \( \mu, \delta, \eta \) and \( \omega \) when a tax is levied in the stock market and in the case of imperfect competition among market makers when \( M = 1000 \) and \( m_s = 0.45 \). The red solid (dashed) line shows the equilibrium ask (bid) price for the benchmark case of no taxation and perfect competition.

This is clear from Figure 8 where we assume that the proportion of dealers in the stock market has decreased to \( m_s = 0.45 \), leading to more dealers providing liquidity in
the option market than in the stock market. In all four graphs of Figure 8 we see that the spread increases compared to the case where just a tax in the stock market is introduced. Recall that introducing a tax in the stock market will lead informed traders to move to the option market, therefore decreasing the level of informed trading in stocks, which in turn has a positive effect on the liquidity.

Imperfect competition on the other hand, while also reducing the amount of informed trading, allows market makers to earn rents by posting higher spreads. The latter effect dominates, resulting in higher spreads compared to the case with taxation in the stock market.

6 Sequential Model

Next we extend the model to allow the dealer to update her beliefs and post new quotes after every trading round, effectively making the model sequential. Given any first-period trade, we can update the market makers beliefs in a straightforward way. As is shown by the probability tree in figure 1 at the beginning of a trading day an information event happens with probability \( \eta \), and, conditional on the information event happening, we are either in a high or low state of the world. Therefore, during the day, the market maker will update her probabilities that an information event happened at the start of the day and her probability of the state of world based on the incoming trades. For example, a stock purchase will increase the market makers belief that we are currently in an up-state, whereas a stock sale or a the purchase of a put option will increase her belief that we are in a down-state. The sequential model can be represented by the following set of equations:

\[
\begin{align*}
\mu_j(\text{Buy Stock}; \mu_{j-1}) &= \frac{\mu_{j-1}(2\alpha_u \delta + \omega - \delta \omega)}{2\alpha_u \delta \mu_{j-1} + \omega - \delta \omega} \\
\mu_j(\text{Buy Call}; \mu_{j-1}) &= \frac{\mu_{j-1}(\delta - 1 + 4(\alpha_u - 1)\beta_u \delta + \omega - \delta \omega)}{\delta - 1 + 4(\alpha_u - 1)\beta_u \delta + \omega - \delta \omega} \\
\mu_j(\text{Sell Put}; \mu_{j-1}) &= \frac{\mu_{j-1}(1 - \omega + \delta(3 + 4\alpha_u(\beta_u - 1) - 4\beta_u + \omega)}{1 - \omega + \delta(4(\alpha_u - 1)(\beta_u - 1)\mu_{j-1} - 1 + \omega)} \\
\eta_j(\text{Buy Stock}; \eta_{j-1}; \mu_{j-1}) &= \frac{\eta_{j-1}(2\alpha_u \delta \mu_{j-1} + \omega - \delta \omega)}{2\alpha_u \delta \eta_{j-1} \mu_{j-1} + \omega - \delta \eta_{j-1} \omega}
\end{align*}
\]
\[ \eta_j (\text{Buy Call}; \eta_{j-1}; \mu_{j-1}) = \frac{\eta_{j-1}(\delta - 1 + 4(\alpha_u - 1)\beta_u \delta \mu_{j-1} + \omega - \delta \omega)}{\delta \eta_{j-1}(1 + 4(\alpha_u - 1)\beta_u \mu_{j-1} - \omega) + \omega - 1} \]

\[ \eta_j (\text{Sell Put}; \eta_{j-1}; \mu_{j-1}) = \frac{\eta_{j-1}(1 - \omega + \delta(4(\alpha_u - 1)\beta_{u-1}\mu_{j-1} + \omega - 1))}{1 - \omega + \delta(4(\alpha_u - 1)\beta_{u-1}\mu_{j-1} + \omega - 1)} \]

The sequential probabilities for the down-state trades are shown in the appendix. Given this set of probabilities, the model provides a time series of trades, quantities and prices.

### 6.1 Option Prices

In order to determine the option prices in the sequential model, we exploit the trinomial structure of stock prices implied by our probability tree. In fact, the model above implies that the stock prices are determined by the following trinomial tree:

\[
S_{t+1} = \begin{cases} 
S_{tu} & \text{with probability } p_u = \eta \times \mu \\
S_{tm} & \text{with probability } p_m = 1 - \eta \\
S_{td} & \text{with probability } p_d = \eta \times (1 - \mu) 
\end{cases}
\]

Note that this structure of stock prices can be seen as the discretized version of stock prices derived from a (hypothetical) geometric Brownian motion. In order to determine the risk neutral probabilities \( \{p_{RNN}, p_{RNm}, p_{RNd}\} \) and the appropriate jump sizes \( \{u, m, d\} \), we construct conditions that match the parameters of the trinomial tree with the first two moments of the distribution of the (hypothetical) geometric Brownian motion, while also imposing that the probabilities are specified so that the growth rate of the stock matches the risk-free rate. We therefore have the following two conditions:

\[ \mathbb{E}[S_{t+1} | S_t] = e^{r\Delta t} S_t \quad \text{(30)} \]

\[ \text{Var}[S_{t+1} | S_t] = \Delta t S_t^2 \sigma^2 \quad \text{(31)} \]

Additionally, we impose the two following constraints on the jumps sizes

\[ ud = 1 \quad \text{and} \quad m = 1 \quad \text{(32)} \]

These constraints imply that the upward jump is the reciprocal of the downward jump,
which leads to a recombining tree, where the number of nodes in the tree grow linearly with the number of steps. Then, if we additionally impose \( p_m^{RN} = 1 - p_u^{RN} - p_d^{RN} \), we end up with three constraints \([30, 31, 32]\) on four parameters \( \{u, d, p_u^{RN}, p_d^{RN}\} \). We therefore have discretion over the choice of the jump sizes, and, following the literature we set:

\[
\begin{align*}
    u &= e^{\beta \sigma \sqrt{\Delta t}}, \\
    d &= e^{-\beta \sigma \sqrt{\Delta t}}
\end{align*}
\]

where \( \beta > 1 \) is chosen such that a risk neutral measure exists (and such that \( p_u^{RN} + p_d^{RN} < 1 \)). The risk neutral probabilities are then given by

\[
\begin{align*}
    p_u^{RN} &= \frac{1 - e^{r \Delta t} - e^{2r \Delta t + \sqrt{\Delta t} \beta \sigma} + \Delta t e^{r \Delta t + \sqrt{\Delta t} \beta \sigma} a^2}{(e^{\sqrt{\Delta t} \beta \sigma} - 1)^2(1 + e^{\sqrt{\Delta t} \beta \sigma})} \\
    p_d^{RN} &= \frac{e^{2\sqrt{\Delta t} \beta \sigma} (e^{r \Delta t} - e^{2r \Delta t} - e^{r \Delta t + \sqrt{\Delta t} \beta \sigma} + \Delta t e^{r \Delta t + \sqrt{\Delta t} \beta \sigma} - \Delta t \sigma^2)}{(e^{\sqrt{\Delta t} \beta \sigma} - 1)^2(1 + e^{\sqrt{\Delta t} \beta \sigma})} \\
    p_m^{RN} &= 1 - p_u^{RN} - p_d^{RN}
\end{align*}
\]

In order to find the option prices, we simply have to determine the option payoffs at maturity \( T \):

\[
\begin{align*}
    C^c(S, T) &= max(S - K_c, 0) \\
    C^p(S, T) &= max(K_p - S, 0)
\end{align*}
\]

The remaining prices can then be determined by the following backward induction formula:

\[
C_{n,t}^{c,p} = e^{-r \Delta t} [p_u^{RN} C_{n+1,t+1}^{c,p} + p_m^{RN} C_{n,t+1}^{c,p} + p_u^{RN} C_{n-1,t+1}^{c,p}] \tag{33}
\]

where \( \{n, t\} \) determine respectively the node and time step of the trinomial tree. In order to make the option pricing coherent with the trading structure shown in Figure 1, we will assume that the step sizes of our trinomial tree \( (t) \) represent days. This implies that the option pricing outlined in this section provides daily option prices for every possible path of our probability tree, which in turn is determined by the set of two probabilities \( \{\eta, \mu\} \). The intradaily option ask and bid prices can then be determined in the following way:
where $k$ defines the trading round during trading day $t$. The ask and bid prices for the put option are found similarly.

6.2 Simulation Results

In this section we show the results of simulating the model described above, that is, the benchmark model without taxation and with perfect competition as well as the model with the inclusion of the different tax regimes and the case of imperfect competition. This exercise allows us to construct time series proxies for volume and liquidity, which will then also be used in the empirical part. We simulate a trading year of data, e.g. 252 days, with 30 trading rounds per day. Figures 9 to 11 show the average results of 1000 simulations for volume and liquidity in different scenarios of the model described previously. Furthermore, the options available for trading are European type options with 4 months (80 days) until expiration, which are rolled over every 3 months (e.g. the options are only available up to one month before expiration).

Figure 9 shows the effect of the different tax regimes on the trading volume in both stock and option markets. The top two graphs show the time series of average quantities summed over one trading day, whereas the bottom two graphs show the dollar weighted volume. We can see that introducing a tax in the stock market significantly reduces the volume in the stock market for both proxies used. Also, introducing a tax in the option market slightly increases the volume in the stock market, but this effect is smaller compared to the effect of a stock tax. This is in line with the asymmetry of a stock versus an option tax that we already discussed in the static version of the model. Given this result, the effect of taxing both markets is straightforward, e.g. the effect of the stock tax dominates. As shown in the right two graphs in Figure 9, the option market is largely unaffected, which is again due to the asymmetric effect of taxation.

Additionally, in the baseline simulation we use in the money options. Compared to the baseline parameters used in the comparative statics of the static model, for the baseline simulation we use $\delta = 0.2$, $S_0 = 100$, $L = 5$, $l = 2$, $\sigma = 0.15$, $\beta = 1$.

The proxies for volume in the sequential model are a function of both trading quantity and trading frequency. The large effect of taxation of stocks on stock trading volume is due to the fact that both
Next, we consider the impact of different tax regimes on market liquidity. In order to do so, we construct four different spread measures, namely quoted spread, effective spread, realized spread and price impact. We will more formally define these proxies in terms of our actual trade and quote data in the empirical section, while for the purpose of this section we limit ourselves to briefly discussing the intuition behind these spreads. The quoted spread is perhaps the most obvious and intuitive measure, and it is simply the cost of a hypothetical "round-trip" transaction, that is, the cost incurred if one would (instantaneously) buy and sell an instrument at the best quotes available. The effective spread on the other hand tries to measure this cost by using the prices actually obtained by investors. It can therefore be seen as the impact of the specific transaction on the prices available in the market.

trading quantities and trading frequency experience large decreases due to taxation. The compound effect is therefore a significant decrease in volume. On the other hand, stock taxation has opposite effects on trading frequencies and trading quantities in the option market. While the former increases, the latter decreases, therefore canceling each other out. Figures for trading frequencies and quantities are provided in the appendix.
Figure 10: The Figure shows the effect of the different tax regimes on the quoted spread, effective spread, realized spread and price impact in the stock market.

Importantly, the effective spread can be decomposed into the realized spread and the price impact, where, in empirical applications, the former is usually interpreted as a proxy of rents obtained by market makers, and the latter is seen as a proxy for the adverse selection present in the markets. Figure 10 shows the effects of the different tax regimes on these spreads for the stock market. The following picture emerges: introducing a stock tax in our setting decreases (increases) the proportion of informed trading in the stock market (option market) which positively affects its liquidity. This can be seen both for quoted and effective spread. Furthermore, it is clear from the two bottom graphs of Figure 10 that this effect is purely driven by the alleviation of the adverse selection problem in the market. Further, introducing a tax in the option market has the opposite effect, even though the magnitudes are again substantially lower compared to introduction of a stock tax. The results for the spreads in the option market (see Appendix) follow the same logic and therefore move in the expected direction, but coherently with the results found so far, are much lower in magnitude.

Next we simulate our model under imperfect competition. As mentioned earlier, we do
Figure 11: The Figure shows the effect of imperfect competition on the different spreads compared to the benchmark case without taxation and with perfect competition and the case where a stock tax is introduced half way through the trading year. The imperfect competition case assumes $M=1000$ and $m_s$ drops from 0.5 to 0.45.

not endogenize the choice of $m_s$, but it is reasonable to assume that the introduction of taxation in a certain market will, if anything, have a negative effect on the competition of market makers in that market. For the simulation with imperfect competition we therefore assume that taxation will also lead to worse competition among market makers by decreasing (or increasing in case of an option tax) the proportion of market makers ($m_s$) that provide liquidity. Figure 11 shows the result of this exercise, specifically the effect of decreasing $m_s$ from 0.5 to 0.4 on spreads compared to the benchmark case and the scenario in which a stock tax is introduced. Figure 11 clearly shows that decreasing the level of competition adversely affects the spreads in the markets. The bottom two graphs of Figure 11 shows that both the quoted spread and the effective spread increase as the proportion of active market markers in the stock market decreases. Opposite to the case where the

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Footnote: Most transaction taxes, as we discuss in section 7, include some sort of exemption for market makers. These exemption vary across countries and can effectively include significant bureaucratic burdens for market makers in order to provide proof of market making activity. Thus, even though exemptions apply, financial taxes can lead to significant indirect costs for market makers.
introduction of a tax does not affect the competition among market makers, this result is mainly driven by an increase in the realized spread. Decreasing competition also positively affects the adverse selection in the market. In fact, as market conditions worsen due to the decreasing competition, informed traders move to the more competitive market.

To summarize, the results from our theoretical model suggest the following. Introducing the same tax rate and/or function in both stock and derivative markets leads to an asymmetric effect of taxation across markets. Taxing stock market significantly affects the volume in the market, which is both an effect of decreased trading quantities as well as lower trading frequencies. Liquidity, as measured by four different spread measures, is affected positively in the market that is taxed. This result arises because of the alleviation of the adverse selection problem as informed trading moves to the untaxed market. When a worsening of the liquidity provision due to taxation is included, the direction of the effect of taxation on liquidity is ambiguous and depends on the magnitude of the two mechanisms at play. In the next section, we test these hypotheses using the recent introduction of financial transaction taxes in France, Spain and Italy.

7 Natural Experiments

Before looking at the specific institutional settings of the financial transaction taxes introduced in Italy, France and Spain, it is worthwhile briefly summarizing the efforts that have been made on a European level over the most recent history in terms of FTT’s.

In the aftermath of the great financial crisis of 2007/08, many countries shared the same common sentiment of holding the financial sector accountable for the costs it had imposed on governments and tax payers. This common sentiment was then formally stated in a request by the G-20 leaders towards the IMF to "prepare a report with regard to the range of options countries have adopted or are considering as to how the financial sector could make a fair and substantial contribution toward paying for any burden associated with government interventions to repair the banking system". This report, among others things, also considered the use of a "Financial Activities Tax" as a form of contribution from the financial sector. Despite these recommendations, efforts to work on the implementation of a form of transaction tax on a G-20 level failed, which prompted the European Union’s executive to work unilaterally on a European wide "Tobin-style" tax. This led to a first proposal, issued in September 2011, in which the European Commission discussed the pos-
sibility of a financial transaction tax applied to the 27 Member States of the Union. The reasons for the introduction of such a tax, as stated in the proposal, can be summarized in three main points: 1.) Ensure the fair contribution of the financial sector to covering the costs of the financial sector as well as leveling the field in terms of financial burden compared to other sectors 2.) Avoiding a possible fragmentation of markets due to numerous uncoordinated national approaches 3.) Discourage risky trading activities that could create competitive distortions and complement regulatory measures aimed at stabilizing the financial system. While this proposal did not garner the required unanimous vote from all the member states, a number of Member States expressed a strong willingness to continue working on a European FTT. This effort translated into a final authorization by the European Parliament to allow 11 member States to engage in the procedure of ”enhanced cooperation” on a common FTT harmonised amongst themselves. Since then, two new proposals have been put forward, one by the European Commission in 2013 and a more recent one, in December 2019, by the German Finance Minister. Finally, in February 2021, the Portuguese Presidency of the Council of the EU proposed an inclusive discussion among all Member States on tax design issues of the European FTT. To the best of our knowledge, these efforts have been the last updates on the discussion of this topic, and no further announcements have been made on the future of this initiative.

Besides the efforts made on a European level, various countries have unilaterally introduced FTT’s during this period, among which Italy, France and most recently Spain. In the next section we will discuss the specific tax design implemented in these countries.

### 7.1 Italian FTT

The Italian FTT was introduced by the Law n. 228/2012, which was officially published in the Gazzetta Ufficiale on the 29. December 2012. The Law formalized the introduction of a tax on stocks and derivative contracts written on these stocks, which was respectively introduced on March 1st and September 1st 2013. More specifically, starting on March 1st, transactions - executed on a regulated markets or MTFs - of shares issued by companies having their registered office in Italy with a market capitalization above €500 million were taxed at a tax rate of 0.1%\(^{17}\). Additionally, a tax rate of 0.2% was applied to all transactions of Italian shares, irrespective of their market capitalization, for trades executed on OTC markets. The tax applies irrespective of the place where the transactions are executed.

\(^{17}\)This also includes securities representing italian shares, such as ADRs, GDRs and EDRs.
and the state of residence of the counterparties. The tax was also extended to include high frequency trading in Italian equities. The tax rate applied to derivative products is computed based on a fixed sliding scale applied to the notional value of the contract, and ranges from €0.01875 to a maximum of €200. Additionally, the rate is reduced to 20 percent of the amount due for transactions executed on regulated markets or MTFs. The taxable basis for transactions of shares is constituted by the net daily balance of taxable transactions and the tax is due by the buy side of the transaction\footnote{This effectively means that intraday transactions are exempted, as long as they net out to 0 at the end of the day.}. For derivatives, the tax is due by both counterparties and the tax is applied to the notional value of the contract. The laws also includes a number of exclusions and exemptions, listing all of which goes beyond the scope of this paper. Importantly though, even though transactions executed in the course of ”market-making activities” are exempted, this exemption is very narrow compared to the French and Spanish legislation.

\subsection*{7.2 French FTT}

The financial transaction tax in France was first announced on February 29th 2012 and came in effect on 1st August of the same year. Compared to the Italian case, the tax in France had a narrower scope, as it was only applied to transactions of equity shares issued by companies headquartered in France with a market capitalization above €1bn. Furthermore, as for Italy, the legislation also included transactions executed over-the-counter in the scope of the tax. Differently from the Italian application of the tax, the same marginal tax rate was applied to both regulated and non regulated markets and transactions of derivative products with french stocks as the underlying were effectively exempted from taxation. The taxable basis, on which a marginal tax rate of 0.2\% was applied\footnote{The original tax rate announced by the french government was 0.1\%, which was then increased to 0.2\% on July 4th 2012. The tax rate was then further increased to 0.3\% on 1st January 2017.}, consists of the acquisition value of equities. That is, as is the case in Italy, the tax is economically borne by the buyer side of the transaction. Additionally, if multiple trades on the same equity are executed during the same day, the net position is considered as the taxable basis. The French tax policy also included various exemptions, most notably one that excludes market makers from the application of the tax\footnote{The law that introduced the FTT contains its own definition of market making. As noted before, this definition is much more generous compared to the Italian case.}.
7.3 Spanish FTT

Finally we look at the most recent introduction of a FTT in a European country, namely the Spanish financial transaction tax, formalized by Law 5/2020 and put into force on January 16, 2021. Similarly to Italy and France, the Spanish tax is levied on acquisitions of shares of Spanish companies listed on a regulated market and with market capitalization greater than €1bn. As in the two previous cases, the tax is applied on a "principle of issue", meaning that the tax is applied irrespective of the tax residence of the parties involved or the location of the transaction. Then, as in France, the tax rate is 0.2%, which is applied to the amount of the consideration for the acquisition net of any transaction fees\(^{21}\). The law furthermore also provides numerous exemptions, where again the most important one for our analysis is the one applied to acquisitions performed during market-making activities.

8 Identification Strategy and data

We leverage the quasi-natural experiment financial transaction tax introductions for stocks and derivatives to test the derived model implications. We adopt a flexible difference-in-differences (diff-in-diff) estimation to identify the different tax impacts on trading activity and market liquidity for three different countries – France, Italy, and Spain. For this purpose, we compare the treated group, stocks whose trading is now taxed, to a untreated group, stocks that are not taxed but that are otherwise as similar as possible.

The control group for France are the Netherlands, for Italy it is Spain and for Spain it is Italy. Both, treated and control stocks are characterized by a market capitalization above the respective threshold as described in section 7.1, 7.2 and 7.3. For both country pairs, namely France/Netherlands and Italy/Spain, the control group is chosen to reflect similar macroeconomic environments and that the respective financial market is comparable with respect to pre-event stock market size, design, and liquidity\(^{22}\). For the analysis of the derivatives market, we use stock options and futures with the treated and control stocks as the underlying.

The stock’s samples consist of all stocks above the country specific threshold and for which consistent data is available. Table 1 shows the number of treated and control stocks, options and futures in detail and table 5 and 6 shows the mean and the standard deviation

\(^{21}\)Again, as in the other cases considered, if multiple trades are executed during one day, the net positive amount is used as the tax base.

\(^{22}\)In the case of France and the Netherlands, Euronext is the main trading venue and therefore trading in French and Dutch stocks follow the same trading protocol, tick size regime, and fee structure.
of the financial market variables. In the case of France, all Euronext stock exchanges are natural candidates. Belgium experienced during the same period a tax increase of a pre-existing financial transaction tax that could affect the impact evaluation of the French FTT. Portugal has been heavily affected by the European sovereign Debt crisis. So we limit the control group to Dutch stocks. In the case of Italy, we chose Spain as control group for its comparable financial market size and exposure to the European debt crisis. Table 5 shows the pre-event stock market summary statistic for trading activity and market liquidity and validates the assumptions.

Table 1: Number of treated and control stocks and derivatives

<table>
<thead>
<tr>
<th></th>
<th>Treated group</th>
<th>Control group</th>
</tr>
</thead>
<tbody>
<tr>
<td>French tax introduction in 2012</td>
<td>81 French stocks above 1 billion EUR</td>
<td>25 Dutch stocks above 1 billion EUR</td>
</tr>
<tr>
<td></td>
<td>2435 stock options on 42 French stocks</td>
<td>3119 stock options on 19 Dutch stocks</td>
</tr>
<tr>
<td></td>
<td>303 futures on 38 French stocks</td>
<td>131 futures on 16 Dutch stocks</td>
</tr>
<tr>
<td>Italian tax introduction 2013</td>
<td>50 Italian stocks above 500 million EUR</td>
<td>51 Spanish stocks above 500 million EUR</td>
</tr>
<tr>
<td></td>
<td>7764 stock options on 20 Italian stocks</td>
<td>9692 stock options on 18 Spanish stocks</td>
</tr>
<tr>
<td></td>
<td>688 futures on 39 Italian stocks</td>
<td>349 futures on 33 Spanish stocks</td>
</tr>
<tr>
<td>Spanish tax introduction 2021</td>
<td>81 Spanish stocks above 1 billion EUR</td>
<td>80 Italian stocks above 1 billion EUR</td>
</tr>
<tr>
<td></td>
<td>9642 stock options on 25 Spanish stocks</td>
<td>16374 stock options on 24 Italian stocks</td>
</tr>
<tr>
<td></td>
<td>205 futures on 29 Spanish stocks</td>
<td>1208 futures on 53 Italian stocks</td>
</tr>
</tbody>
</table>

The option’s samples are constructed as follows. We start defining and event window of 6 months before and after the event. We collect all options which are live during the event window with a stock in the treated or control group as the underlying. For example in the case of France, this gives us 2435 stock options on 42 stocks with a market capitalization above one billion Euros. We drop options by moneyness smaller than 0.9 or greater than 1.1 and also with maturity greater than five trading weeks and smaller than one trading week. Table 6 shows the pre-event stock market summary statistic for trading activity and market liquidity.

The future’s samples are constructed in a similar manner. We compile all futures which are live during the event window with a stock in the treated or control group as the underlying. For the futures, we drop by the same maturity measures as for the options.

In our identification strategy in equation (R.1), we compare all trading activity and...
market quality measures of stocks and derivatives with a market capitalization above the
country specific threshold and thus affected by the tax to the untreated stocks above the
same threshold in the control country. Equation (R.1) shows the formal definition.

Formally, the model described satisfies for each stock/derivative $i$ and date $t$ the equation

$$E(y_{i,t}) = \alpha_i + \gamma_t + \beta D_{i,t}$$  \hspace{1cm} (R.1)

where $D_{i,t}$ is the dummy variables that takes the value of one for treated stocks in the
months after the tax introduction and is zero otherwise. The terms $\alpha_i$ captures the security’s
time-invariant fixed effects and $\gamma_t$ its time fixed effects. Standard errors are clustered
by security and time following Thompson (2011). The regression model specification (R.1),
like all diff-in-diffs, relies on the untestable but crucial common trend assumption. It says,
in absence of the treatment, treated and control group would have co-moved closely. It
became customary in the diff-in-diff literature to use visual inspection to check for the va-
lidity of the control group (see figure 12) and run placebo diff-in-diff estimations. Placebo
diff-in-diff is a replication of equation (R.1) on random event days. In absence of the treat-
ment, regression results should be statistically not different from zero.

The tests confirm
the validity of our control groups.

All empirical tests and results are based on security-day level. We use nanosecond-
stamped intraday trade and quote data from Thomson Reuters Tick History.

8.1 Measures of trading activity and market liquidity

In this section, based on the theoretical model of two markets, we derive the empirical
financial market variables necessary to test the model predictions. We focus on trading
activity and market liquidity measures.

As the model in 4.5 describes, an exogenous increase in trading costs or more directly a tax on trading financial securities changes the propensity to trade that treated security or on that exchange. Therefore, firstly we assess trading activity of stocks and its derivatives by estimating changes in trading volume and frequency of taxed securities. Secondly, we explain market depth and decompose the standard bid-ask spread into liquidity supplying and demanding trading costs.

Log volume is a measure of trading activity and calculated as the natural logarithm of the sum of EUR values traded on day \( t \) and for stock/derivative \( i \).

Trading frequency is the count of trades over a day. It is independent of the number of traded securities and its EUR value. It is one, when a trade happens and zero otherwise.

Log depth is the natural logarithm of the mean of the available liquidity at the inside spread of bid and ask sides.

As the model in 5.1 describes, adverse selection affects spreads and provided liquidity. The standard measure of the cost of a small round-trip transaction is the difference between the best ask and bid quote normalized by the mid-price. It can be read in percentage points or basis points (BPS), depending on the size. It is often understood as the spread. The richness and abundance of Reuters nanoseconds financial market data allows us to unbundle this spread into four more precise spread measures and its quoted depth. Following Foucault, Pagano, Roell, and Röell (2013) we decompose the bid-ask spread into the quoted spread, the effective spread, the price impact and the realized spread.

The quoted spread is the weighted average bid-ask spread from the quotes posted on day \( t \) for security \( i \). The standard bid-ask spread is only a good measure of trading costs for orders that are small enough to be entirely filled at the best quotes. But if the trade size increases, the available liquidity at the inside spread matters for the price impact. The quoted spread is defined as

\[
\text{quoted spread} = \frac{(a_\tau - b_\tau)(q_\tau/q_{i,t})}{m},
\]

where \((a_\tau - b_\tau)(q_\tau/q_{i,t})\) is the weighted average bid-ask spread divided by the mid-price \((m)\).

The effective spread calculates trading costs using the prices actually obtained by investors and measures the slippage. It is defined as the difference between the price at which a market order executes \((p_\tau)\) and the mid-quote \((\text{mid}_\tau)\) on the market the instant right before the trade happens. The effective spread is likely to increase with the size of transactions. The effective spread for trade \( \tau \) for a given security is then given as
Effective spread \( q \tau \frac{p_\tau - mid_\tau}{mid_\tau} \),

where \( q_\tau \) is a buy-sell indicator taking the value of 1 (-1) for buys (sells).

Trades are signed using the algorithm proposed by Lee and Ready (1991). The effective spread is comprised of the realized spread and the price impact. The latter two add up approximately to the effective spread. The realized spread can be interpreted as compensation liquidity providers require for the adverse price movement following a trade. Therefore, it is a proxy for revenues of liquidity provision. The price impact can be interpreted as a measure of adverse selection.

The realized spread implicitly adopts the viewpoint of liquidity suppliers and is calculated as the difference between the transaction price and the mid-price five minutes after the transaction. The underlying assumption is that the interval should be long enough to ensure that market quotes have adjusted to reflect the price impact of the transaction.

\[
\text{realized spread} = q_\tau \frac{p_\tau - mid_{\tau+5min}}{mid_\tau},
\]

where \( mid_{\tau+5min} \) is the mid-quote five minutes after the transaction.

The price impact is defined as follows and measures transaction costs that are based on the extent to which an order generates an adverse reaction in the market price.

\[
\text{price impact} = q_\tau \frac{mid_{\tau+5min} - mid_\tau}{mid_\tau},
\]

9 Results

We start by presenting the results for the introduction of the tax on stocks in Italy, which are shown in table 2 and figure (12). Column (1) shows the effect on volume and market liquidity for stocks traded on regulated markets. We can see that the Italian FTT led to a significant decrease in market liquidity, both in terms of quoted and effective spread. The increase in the latter seems to be driven both by an increase of the realized spread as well as price impact. In terms of volume, even though the estimate is negative, therefore suggesting a decrease in trading activity compared to the control group, the effect is not statistically significant. On the other hand, we find a significant drop in trading.

\footnote{For all our results on stocks traded on regulated markets we use data for the national exchanges, e.g. Borsa Italian (Italy), Euronext Paris (France), Bolsa de Madrid (Spain). Our choice is based on the fact that the national markets account for the vast majority of the liquidity for stocks issued in the respective countries.}
volume for Italian stocks traded in OTC markets (column (4))\textsuperscript{27}. This result is further supported by the quarterly aggregated year-to-year changes of OTC volume in Italian markets reported by the Italian Companies and Exchange Commission (CONSOB) (table 7, see Appendix). Column (2) and (3) report the results for Options and Futures written on Italian Stocks traded on regulated markets. Specifically, we report measures for the aggregation of derivatives traded on the Borsa Italiana and EUREX, which combined account for the vast majority of on-exchange derivatives trading for the products considered. As is evident, we do not find any significant effects on volume or liquidity for both type of derivative products.

\textsuperscript{27}The results are based on Italian stocks traded in US OTC markets, as OTC data for European markets are only available for highly aggregated frequencies.
Figure 12 illustrate the difference-in-difference estimates for the causal impact of the FTT on trading volume on-exchange and OTC. The plots show the cross-sectional average for treated (black) and control (light gray) stocks, minus the respective pre-event averages. The time series are smoothed with a five-day moving average. The dashed lines indicate the averages for the treated and the control group. By construction they are zero and the same before the event. After the event, the difference reflects the diff-in-diff estimate of the regression result in table 2.

Table 8 (see Appendix) shows the results for the introduction of the tax on derivatives, which was introduced 6 months later, on September 1st 2013. We do not find any significant
Table 2: Causal Impact of the Italian FTT - Stock tax

This table presents the estimates for the coefficient $\beta$ from specification (R.1), where the dependent variable corresponds to proxies for volume and liquidity described in section 8.1. The regression is applied to the introduction of the tax on Italian stocks with a market capitalization above €500 million and the event date is therefore 01.03.2013. Standard errors (in parenthesis) are clustered by stock and time and as usual ***, **, * denote statistical significance at the 1%, 5% and 10% level.

<table>
<thead>
<tr>
<th></th>
<th>Stocks</th>
<th>Options</th>
<th>Futures</th>
<th>OTC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trading activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log volume</td>
<td>-0.057</td>
<td>-0.121</td>
<td>0.094</td>
<td>-0.600***</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
<td>(0.182)</td>
<td>(0.289)</td>
<td>(0.219)</td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log depth</td>
<td>-0.040</td>
<td>-0.188</td>
<td>0.354</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.137)</td>
<td>(0.243)</td>
<td></td>
</tr>
<tr>
<td>quoted spread</td>
<td>0.484**</td>
<td>-0.001</td>
<td>-0.003*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.199)</td>
<td>(0.008)</td>
<td>(0.002)</td>
<td></td>
</tr>
<tr>
<td>effective spread</td>
<td>0.214***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.081)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>realized spread</td>
<td>0.153**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>price impact</td>
<td>0.034*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

# treated: 51 6840 223 65
# control: 50 6407 125 37
# observations: 8200 108820 5205 5305

changes neither in volume nor liquidity, suggesting that the tax on derivatives either had a very mild effect on the markets considered or that the effect had already been anticipated by markets in the introduction of the tax on stocks.

Table 3 and 4 show the results for the introduction of the tax on respectively French and Spain stocks with market capitalization above 1 billion EUR. As described previously, the tax design in France and Spain, as opposed to the one in Italy, are very similar, which is mirrored in our findings. For both tables column (1) reports the result for stocks traded on exchange, and for both countries we find a significant drop in volume. For Spain we see that the volume decreased by 28% compared to the control group, which is three times as much as the reduction of volume we find for France. Compared to the Italian FTT, we do not find any significant drop in OTC volume and market liquidity seems to be largely unaffected. Similar to Italy on the other hand, trading in the derivative products of the taxed stocks, at least for on exchange trading, is no affected by the introduction of a transaction tax on the underlying.
Table 3: Causal Impact of the French FTT

This table presents the estimates for the coefficient $\beta_{permanent}$ from specification (R.1) (Note that for France we use the flexible model described in footnote 20) where the dependent variable corresponds to proxies for volume and liquidity described in section 8.1. The regression is applied to the introduction of the tax on French stocks with a market capitalization above €1 billion and the event date is therefore 01.08.2012. Standard errors (in parenthesis) are clustered by stock and time and as usual ***, **, * denote statistical significance at the 1%, 5% and 10% level.

<table>
<thead>
<tr>
<th></th>
<th>Stocks</th>
<th>Options</th>
<th>Futures</th>
<th>OTC</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trading activity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log volume</td>
<td>-0.093**</td>
<td>-0.473</td>
<td>-0.259</td>
<td>-0.019</td>
</tr>
<tr>
<td></td>
<td>(0.057)</td>
<td>(0.321)</td>
<td>(0.328)</td>
<td>(0.278)</td>
</tr>
<tr>
<td><strong>Liquidity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log depth</td>
<td>-0.007</td>
<td>-0.167</td>
<td>4.618***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.194)</td>
<td>(0.872)</td>
<td></td>
</tr>
<tr>
<td>quoted spread</td>
<td>-0.091</td>
<td>-0.001</td>
<td>-0.019*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.585)</td>
<td>(0.216)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>effective spread</td>
<td>0.221</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.156)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>realized spread</td>
<td>0.002</td>
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<tr>
<td></td>
<td>(0.085)</td>
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<tr>
<td>price impact</td>
<td>0.124</td>
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</tr>
<tr>
<td></td>
<td>(0.098)</td>
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</tbody>
</table>

| # treated           | 81       | 1036     | 112     | 120      |
| # control           | 25       | 1260     | 43      | 37       |
| # observations      | 11818    | 34113    | 4257    | 12127    |

9.1 Discussion

The results described in the previous section have various implications in terms of the (i) theoretical predictions established in the first part of the paper, (ii) the specific tax designs of each country and (iii) previous findings by the literature. As we mentioned previously, the tax design in Italy differs significantly from the design implemented in France and Spain across various dimensions. Firstly, a differential tax rate was introduced for stocks traded on exchange and on unregulated markets. As we saw, this led to a significant reduction in trading volume in the OTC market, whereas trading volume executed on exchange was statistically unaffected. From the perspective of our model, this differential tax rate for two different markets with (virtually) the same products (e.g. stocks of Italian companies) is akin to trading stocks in two separate markets (as opposed to a stock and an option market). Intuitively, if a differential tax rate is introduced - and given that the same product is traded in both markets - this would move trading from the market with the higher tax rate to the market with the lower tax rate, therefore decreasing volume
Table 4: Causal Impact of the Spanish FTT

This table presents the estimates for the coefficient $\beta$ from specification (R.1) where the dependent variable corresponds to proxies for volume and liquidity described in section 8.1. The regression is applied to the introduction of the tax on Spanish stocks with a market capitalization above €1 billion and the event date is therefore 16.01.2021. Standard errors (in parenthesis) are clustered by stock and time and as usual ***, **, * denote statistical significance at the 1%, 5% and 10% level.

<table>
<thead>
<tr>
<th></th>
<th>Stocks</th>
<th>Options</th>
<th>Futures</th>
<th>OTC</th>
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<td>-0.284***</td>
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<td>(0.101)</td>
<td>(0.060)</td>
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<tr>
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<td>-0.624***</td>
<td>-0.698*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.128)</td>
<td>(0.357)</td>
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<tr>
<td>quoted spread</td>
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<td>-0.010*</td>
<td>0.001**</td>
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<tr>
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<td>(0.070)</td>
<td>(0.019)</td>
<td>(0.001)</td>
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<tr>
<td>effective spread</td>
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<td></td>
<td>(0.030)</td>
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<td>realized spread</td>
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<td>price impact</td>
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</table>

in the former and increasing it in the latter. Empirically, the former result holds, whereas the volume in the market with the lower tax rate (regulated markets) is not affected. This could be due to a few different mechanisms. For once, there could be alternative products, for which data is not available, that are used at least by some traders as substitutes for stocks. One type of products that intuitively would lend themselves as possible substitutes are derivatives traded in OTC markets. Such products, opposed to derivatives traded on exchange, are typically non-standardized and are generally able to replicate a much larger set of payoff structure compared to standardized derivatives traded on exchange. On the other hand, the reason we do not see an increase in trading volume for on-exchange trades could be due to an overall drop in volume for Italian stocks.

The results found for the effect on the liquidity of stock markets are very different compared to the findings by other articles that analyzed the Italian FTT, such as [Cappelletti, Guazzarotti, and Tommasino (2017)] and [Hvozdyk and Rustanov (2016)], which do not find any significant effects using daily data. As shown in table 2 using high frequency data we find significant increases in both quoted spread and effective spread, suggesting that the
Italian FTT did indeed worsen the liquidity of stock markets. In terms of our theoretical predictions, this suggests that Italy experienced a decrease in competition among market makers, leading to higher spreads on average. This is supported by the fact that the driving component of the increase in effective spread is the realized spread, which is connected to increased costs of liquidity provision. Additionally, this result is also consistent with the narrow market making exemption used in Italy. As obtaining an these laborious, the cost of liquidity provision was indirectly affected by taxation. Our results suggest that this led to a proportion of market maker exiting the market, therefore reducing competition among liquidity providers. Also, we do not find a decrease in price impact due to less informed trading, as predicted by our model when a stock and option market is considered. Given the (additional) differential tax for regulated and non-regulated market and the consequent reduction in OTC volume we discussed, it is reasonable to assume that potential informed trading previously active in OTC markets moved to regulated markets, therefore counteracting any movement of informed traders active on regulated markets towards other type of products.

The results we find for France are consistent with the aggregate results found by Colliard and Hoffmann (2017). Again, consistent with our theoretical predictions, we find a significant decrease in volume. On the other hand, we do not find any evidence of decreased OTC volume, suggesting that an equal tax rate in regulated and unregulated markets will disproportionately affect the former (e.g. the more liquid market). The same holds for the newly introduced tax in Spain, which as we described previously, shares very similar features to the French FTT. Additionally we do not find any aggregate effects on market liquidity

28 As Colliard and Hoffmann (2017) note, aggregate results might mask differential impacts between stocks with different liquidity levels. As we are not in possession of granular data, we are not able to effectively discern between stocks of different liquidity levels.
traded on the same market, leaving aggregated liquidity largely unaffected. On the other hand, as mentioned earlier, our results could also point towards a potential substitution in derivative products traded in OTC markets, for which we are unfortunately not able to obtain data.\footnote{Over the last decades efforts have been made to make European OTC derivative markets more transparent. As of 2012, EU entities that engage in derivatives transactions are required to report their trades to trade repositories. To the best of our knowledge, these data is only available to regulators.} It is left for future research to discern the magnitude of these two effects.

\section*{9.2 Policy Implications}

The most recent impact of COVID-19 on public finances is unprecedented. In many countries, the ratios of government debt to GDP have reached all-time highs.\footnote{Total public debt as percent of gross domestic product for the US amounted to 120\% \cite{fred}, 101\% for the European Union, 150\% for Italy, 125\% for Spain and 120\% for France \cite{eurostat}.} Moreover, the commitment to the Paris Agreement implies shifting public expenditures to the green transition in the near future. Governments are in dire need of additional facilities to increase taxes.\footnote{As pointed out by Dávila \cite{davila}, the collapse of the Bretton Woods system motivated James Tobin's well-known 1972 proposal to throw sand in the wheels of financial markets, the Black Monday (1987) motivated Stiglitz (1989) and Summers and Summers (1989) to argue for a transaction tax and most recently the great financial crisis brought a tax proposal by the European Commission and tax introductions in France and Italy.} Periodically after economic crisis, taxation of financial transactions to support public finances gains broad relevance on the political stage. In this section we will outline the advantages and disadvantages of a financial transaction tax based on the theoretical and empirical findings of our paper.

The contribution of the FTTs in its current designs for public finances is modest at best. The French FTT collected in 2020 1.8 billion Euros (0.1\% of GDP), the UK stamp duty 4.4 billion (0.2\%) and the Italian FTT 400 million (0.2\%). Our findings suggest, that this is due to various reasons. First, the broad list of exemptions reduces the taxable base drastically in all jurisdictions. Casually speaking, after exempting purchases by market makers, financial intermediaries as liquidity suppliers or responsible for price stabilization, CCPs and CSD, there is almost nothing left to tax. Additionally, there is the behavioral response to the tax introduction. For France, a tax of 10 bps reduces trading volume by 10-20\% and for Spain a tax of 20 bps reduces the volume by 20-30\%.

Second, as the findings of the Italian case allows to deduce, there are incentives to migrate trading to cheaper venues if possible. The theoretical model predicts a weaker reaction of derivatives to a FTT introduction relative to stock markets. Our empirical results confirm the prediction. We do not find a synthetic reproduction of stocks on its
on-exchange derivative markets. Through talks with sell-side equity-trading specialists, we gather anecdotal evidence that in the case of France, the preferred avoidance strategy of the FTT has become contract-for-differences (CFDs).\footnote{CFD are equity derivative contracts that mimic the underlying stock. The main difference between a CFD and stocks is that with CFDs one never owns the stock because it is never bought or sold. CFDs provide on-to-one the up- and down-side potential of the underlying. CFDs are leveraged products and are able to reveal therefore a different risk profile than stocks. CFDs are exempt from the stamp duty in the UK.} The French financial markets regulator (AMF) is closing this loophole through restriction of the marketing, distribution or sale of CFDs to retail investors \footnote{At the moment, only Italy covers on-exchange stock trading, its OTC market and its on-exchange derivative market. It currently does not include OTC derivatives.} \cite{AMF2019}. This suggest that an optimal financial transaction tax rate should target on-exchange stock trading, its OTC market and its derivative market on-exchange and OTC.\footnote{34} As our theoretical model suggest, the derivative tax might not have a fundamental impact but it discourages efforts to find loopholes in the derivative markets.

Thus, our analysis suggests that, in order to maximize tax revenue while minimizing liquidity impacts, a pan-European tax should be preferred over unilateral, national taxes. The widespread coverage of such a tax would allow for less exemptions, as the incentive of avoidance for liquidity providers are reduced. Additionally, derivative markets should be included in the scope of the tax and taxed a tax rate/function that takes into account the specific payoff structure of the instrument.

\section{Conclusions}

We study how asymmetric information and taxation affect trading activity and market liquidity in a multi-market setting. Theoretically, we derive a model of informed and liquidity trading in stock and its option market under adverse selection and taxation of transactions in neither, either or both markets. Additionally, we include possible effects of taxation on liquidity provision (e.g. imperfect competition among market makers) to enrich the model beyond adverse selection effects. Empirically, we leverage quasi-random experiments where only stock and OTC trading is taxed and derivatives are exempt, but also where on-exchange, OTC and derivative markets are taxed. For causal inference, we compare the treated to untreated control securities in a diff-in-diff framework.

We show three main results. First, we find an asymmetric responses of stock versus derivative taxation. Taxing stock markets has a stronger effect on trading activity than taxing derivative markets if the same tax rate is applied. Second, tax design has a significant
effect on outcomes in terms of volume and market liquidity. While Italy experienced a significant reduction in liquidity without any effects on volume, stock markets in France and Spain experienced the opposite effect. In light of our theoretical predictions, we are able to argue that these effects are largely due to different implementations of the tax policy. Third, while our results are limited to regulated markets, we find no evidence for the often expressed possibility to replace stocks by synthetic derivatives in order to avoid taxation.

Finally, our results have potential policy implications. Under the premise that a FTT is introduced to support public finances, policy makers have to strictly focus on the tax design and particularly on the tax exemptions. As we see with the cases of France/Spain in comparison to Italy, the tax design and its exemptions crucially dictates its success. Further, under the premise that the "financial sector could make a fair and substantial contribution toward paying for any burden associated with government interventions to repair the banking system" (IMF 2009), policy makers have to reassess its exemptions for the major market participants as currently the tax burden is mostly passed through to the non-financial market participants. The contribution of the financial sector is in that case neither fair nor substantial.
References

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Coelho, Maria, 2016, Dodging robin hood: Responses to france and italy’s financial transaction taxes, Available at SSRN 2389166.


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FRED, Economic Data, 2021, Federal debt: Total public debt as percentage of gross domestic product.


11 Mathematical appendix

11.1 Derivation of Quantities in Lemma 1

Since $U(x) = x - \gamma x^2$, the objective function for liquidity trader type 1 who trades in the stock market is

$$\max_{\{Q_s\}} \mathbb{E}[U(Q_s)] = \max_{\{Q_s\}} \left[ \eta\mu(-L + (S_0u - A_s)Q_s) - \eta\mu\gamma((-L + (S_0u - A_s)Q_s)^2 \right]$$

$$+ (1 - \eta)(-l + (S_0m - A_s)Q_s) - (1 - \eta)\gamma((-l + (S_0m - A_s)Q_s)^2$$

$$+ \eta(1 - \mu)((S_0d - A_s)Q_s) - \eta(1 - \mu)\gamma((S_0d - A_s)Q_s)^2$$

Then, taking the FOC with respect to $Q_s$ and setting it equal to 0 gives us the following equation:

$$\max_{\{Q_s\}} \mathbb{E}[U(Q_s)] = (A_s - d)(-1 + \mu) + 2(A_s - d)^2 Q_s \gamma(-1 + \mu)$$

$$+ (-A_s + u)\mu - 2(L + Q_s(A_s - u))(A_s - u)\gamma\mu = 0$$

which can be solved to obtain $f_{Qs1}$:

$$f_{Qs1} = \frac{d - d\mu + u(1 + 2L\gamma)\mu - A_s(1 + 2L\gamma\mu)}{2\gamma(A_s^2 + 2A_s d(-1 + \mu) - d^2(-1 + \mu) - 2A_s d\mu + u^2\mu);$$

where we already assumed $\eta = 1, m = 1, K_c = S_d$ and $K_p = S_u$.

Similarly, the objective functions for liquidity traders type 1 who trades call or put options are

$$\max_{\{Q_c\}} \mathbb{E}[U(Q_c)] = \max_{\{Q_c\}} \left[ \eta\mu(-L + ((S_0u - K_c)^+\theta - A_c)Q_c)$$

$$- \eta\mu\gamma((-L + ((S_0u - K_c)^+\theta - A_c)Q_c)^2$$

$$+ (1 - \eta)(-l + ((S_0m - K_c)^+\theta - A_c)Q_c)$$

$$- (1 - \eta)\gamma(-l + ((S_0m - K_c)^+\theta - A_c)Q_c)^2$$

$$+ \eta(1 - \mu)((S_0d - K_c)^+\theta - A_c)Q_c$$

$$- \eta(1 - \mu)\gamma(((S_0d - K_c)^+\theta - A_c)Q_c)^2,$$
\[
\begin{align*}
\max_{\{Q_p\}} \mathbb{E}[U(Q_p)] &= \max_{\{Q_p\}} \left[ \eta \mu (-L + (B_p - (K_p - S_0 u)^+ \theta)Q_p) \right. \\
&\quad - \eta \mu \gamma (-L + (B_p - (K_p - S_0 u)^+ \theta)Q_p)^2 \\
&\quad + (1 - \eta)(-l + (B_p - (K_p - S_0 m)^+ \theta)Q_p) \\
&\quad - (1 - \eta)\gamma (-l + (B_p - (K_p - S_0 m)^+ \theta)Q_p)^2 \\
&\quad + \eta(1 - \mu)(B_p - (K_p - S_0 d)^+ \theta)Q_p \\
&\quad - \eta(1 - \mu)\gamma ((B_p - (K_p - S_0 d)^+ \theta)Q_p)^2 \right] \\
\end{align*}
\]

Again, we can take the FOC with respect to \(Q_s\) and \(Q_p\) and set it equal to zero, which gives us the following equations:

\[
\frac{\partial \mathbb{E}[U(Q_c)]}{\partial Q_c} = A_c(-1 + \mu) + 2 A_c^2 Q_c \gamma (-1 + \mu) - (A_c + (d - u)\theta)\mu \\
- 2 \gamma (A_c + (d - u)\theta)(L + Q_c(A_c + (d - u)\theta))\mu = 0,
\]

\[
\frac{\partial \mathbb{E}[U(Q_p)]}{\partial Q_p} = B_p + (d - u)\theta + 2 Q_p \gamma (B_p + (d - u)\theta)^2 (-1 + \mu) \\
- 2 B_p(-L + B_p Q_p)\gamma \mu + (-d + u)\theta \mu = 0
\]

Solving these equations gives us \(f_{QC1}\) and \(f_{QP1}\):

\[
f_{QC1} = \frac{A_c + 2c L \gamma \mu + (d - u)(1 + 2L \gamma)\theta \mu}{2 \gamma (A_c^2(-1 + \mu) - (A_c + (d - u)\theta)^2 \mu)},
\]

\[
f_{QP1} = \frac{(d - u)\theta(-1 + \mu) - B_p(1 + 2L \gamma \mu)}{2 \gamma ((B_p + (d - u)\theta)^2(-1 + \mu) - B_p^2 \mu)}
\]

The same procedure can be followed for the type 2 liquidity traders that trade in the stock and the option market and obtain \(f_{QS2}, f_{QC2}\) and \(f_{QP2}\):

\[
f_{QS2} = \frac{B_s - 2 B_s L \gamma (-1 + \mu) + d(1 + 2L \gamma)(-1 + \mu) - u \mu}{2 \gamma (B_s^2 + 2 B_s d(-1 + \mu) - d^2(-1 + \mu) - 2 B_s u \mu + u^2 \mu)},
\]

\[
f_{QC2} = \frac{B_c - 2 B_c L \gamma (-1 + \mu) + (d - u)\theta \mu}{2 \gamma (B_c^2 + 2 B_c (d - u)\theta \mu + (d - u)^2 \theta^2 \mu)},
\]

55
\[ f_{Q_2} = \frac{(u-d)(1+2L\gamma)\theta(\mu - 1) - A_p(2L\gamma(\mu - 1) - 1)}{2\gamma((A_p + (d-u)\theta)^2(-1 + \mu) - A_p^2\mu)}. \]

### 11.2 Proof of Proposition 2

We are seeking a linear rational expectation equilibrium, where we assume that the quantities provided and the price schedule are linear functions of the price and quantities respectively:

\[ Q^m_s(p) = \phi_s p \quad \text{and} \quad p_s^*(q) = \lambda_s Q_s \]

Given these strategies, we can compute the residual supply function for a single market maker in the stock market:

\[ Q_s = q^m_s m_s + (M - 1)\phi_s p m_s \quad \Rightarrow \quad p_s = \frac{Q_s - q^m_s m_s}{(M - 1)\phi_s m_s} \]

Now we assume that the market maker knows the market order \( q \) (or alternatively that he infers the total market order \( q \) from the market price), and therefore that he valuates the asset as \( \mathbb{E}[S \mid Q_s] = \alpha_s Q_s \), where (for \( Q_s > 0 \))

\[ \alpha = \frac{(m - m\eta + d(-1 + \delta)\eta(-1 + \mu)\omega + u\eta\mu(2\alpha_u\delta + \omega - \delta\omega)}{2\alpha_u\delta\eta\mu + \omega - \delta\eta\omega} \]

The market maker will therefore choose his quantity schedule to maximize:

\[ \max_{\{q^m_s\}} [p_s - \alpha_s Q_s] \quad \Rightarrow \quad \max_{\{q^m_s\}} \left[ \frac{Q_s - q^m_s m_s}{(M - 1)\phi_s m_s} - \alpha_s Q_s \right] \]

The first-order condition is then

\[ \frac{Q_s - 2q^m_s}{(M - 1)\phi_s m_s} - \alpha_s Q_s = 0 \]

and therefore

\[ q^m_s = \frac{Q_s}{2m_s} \left[ 1 - \alpha_s(M - 1)\phi_s m_s \right] \]

and

\[ p_s = \frac{Q_s}{2m_s} \left[ \frac{1}{(M - 1)\phi_s} + \alpha_s \right] \]

and finally we can eliminate \( Q_s \) from these two equations:
\[ Q_s^m(p) = \frac{(M-1)\phi_s[1 + \alpha_s(M-1)m_s\phi_s]}{1 + \alpha_s(M-1)\phi_s} p_s \]

Then, if we assume a symmetric equilibrium where all market makers use the same supply function, we have

\[ \phi = \frac{(M-1)\phi_s[1 + \alpha_s(M-1)m_s\phi_s]}{1 + \alpha_s(M-1)\phi_s} \implies \phi = \frac{(M-2)}{(M-1)(1 - m_s + Mm_s)\alpha_s} \]

Due to the market clearing condition we then have

\[ \sum_{t=1}^{M} \phi_s p_s m_s = Q_s \implies p^* = \frac{(M-1)(1 - m_s + Mm_s)\alpha_s}{M(M-2)m_s} Q_s \]

And therefore we have that, in equilibrium, market makers in the stock markets will post a pricing schedule equal to

\[ Q^m_s(p) = \phi_s p \quad \text{with} \quad \phi_s = \frac{(M-2)}{(M-1)(1 - m_s + Mm_s)\alpha_s} \]

and the equilibrium price is given by

\[ p_s(p) = \lambda_s Q_s \quad \text{with} \quad \lambda_s = \frac{(M-1)(1 - m_s + Mm_s)\alpha_s}{M(M-2)m_s} \]

Note that, as \( M \to \infty \), we obtain the same results as in the benchmark case. For the option market the same logic can be applied, and we obtain the following result:

\[ Q^m_o(p) = \phi_o p \quad \text{with} \quad \phi_o = \frac{(M-2)}{(M-1)(Mm_s - m_s - M)\alpha_o} \]

\[ p_o(p) = \lambda_o Q_o \quad \text{with} \quad \lambda_o = \frac{(M-1)(Mm_s - m_s - M)\alpha_o}{M(M-2)(1 - m_s)} \]

11.3 Asymmetric Effect of tax in stock vs. option market

The asymmetric effect of taxation across stock and derivative markets, that is, the effect shown for example in figure 5, where we can see that the equilibrium \( \alpha \)'s are much more affected by a taxation on stocks compared to a taxation on options, arises in the following
way in our model. Firstly, equilibrium $\alpha$’s are found by equalizing profits across stock and option markets, and profits are a function of both quantities and prices. Since Prices are not directly affected by taxation (in equilibrium they are indirectly affected by taxation through the asymmetric effect needs to come from the effect of taxation on quantities. Quantities are found by maximization the Utility of terminal wealth of liquidity traders, which is a function of their payoffs in the the different states, as shown in equations [3-6].

![Diagram 1]

Figure 13: The figure shows the (net of the transaction price) profit of buying a stock (dark and light grey area) versus buying a call. The dark grey area represents the taxed part of the payoff. We use a tax of $t_s = t_o = 0.04$. The upper two graphs show the payoffs for $\theta = 1$, lower two graphs show the payoff for $\theta = 5$

The utility of LT’s in the stock market is affected (shifted downwards) way more by
a tax compared to the utility of a LT that trades in the option market when affected by a tax on options. This means that LT’s in the stock market adjust their strategies more when faced by a tax compared to LT’s that trade in the option market. The reason for this effect is that, when the same tax function and rate is introduced in both markets, the taxed proportion of the payoff of investing in either market will be much smaller for trades in the option market. This can clearly be seen in figure 13. This differential impact of taxation on quantities ultimately means that profits (for informed traders) in the stock market will be more affected by taxation on stocks compared to profits in the option market, which then leads to the effect on $\alpha^*$’s shown in figure 5 when the profits in both market are equalized in the presence of taxation.

The intuition behind the differential effect of taxation on quantities is that options provide inherently larger payoffs relative to the amount invested compared to stocks. To the extent that standard taxation affects the amount invested, and since the amount invested relative to the potential payoff for derivatives is ”small” compared to the stock market, the option trader will adjust his strategy less compared to the stock trader when affected by the same tax.
11.4 Additional Figures - Static Model

Figure 14: The Figure shows the equilibrium quantities traded in the option market as functions of $\eta$, $\delta$, $\theta$ and $L$. The red dashed line shows the equilibrium quantities traded in the stock (option) market when there is no tax.
Figure 15: The Figure shows the equilibrium bid prices for the stock as functions of $\mu$, $\delta$, $\eta$ and $\omega$. The dashed line shows the equilibrium bid prices in the stock market when there is no tax.
Figure 16: The Figure shows the histogram of daily frequencies for the different tax regimes. The dark bins show the frequency for when the tax is in place, the light bins show the frequency without taxation.
## 12 Empirical appendix

Table 5: Summary statistics of stock market variables

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<td>1.83</td>
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<td>1.58</td>
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<td>2.83</td>
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<td>1.01</td>
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<td>1.14</td>
<td>2.47</td>
<td>2.94</td>
<td>3.13</td>
<td>2</td>
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<td># stocks / derivatives</td>
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<td>25</td>
<td>50</td>
<td>51</td>
<td>81</td>
<td>80</td>
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<td>3321</td>
<td>1025</td>
<td>2050</td>
<td>2550</td>
<td>4212</td>
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<td>76</td>
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<td># observations</td>
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<td>2125</td>
<td>2301</td>
<td>1092</td>
<td>3936</td>
<td>4669</td>
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### Table 6: Summary statistics of equity derivative market variables

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<th>Option trading activity</th>
<th>France</th>
<th>Netherlands</th>
<th>Italy</th>
<th>Spain</th>
<th>Spain</th>
<th>Italy</th>
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<tr>
<td>log volume</td>
<td>3.88</td>
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<tr>
<td>(1.73)</td>
<td>(2.17)</td>
<td>(2.65)</td>
<td>(1.96)</td>
<td>(1.99)</td>
<td>(1.81)</td>
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<td>Stock liquidity</td>
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<td>9.54</td>
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<tr>
<td>(1.80)</td>
<td>(2.36)</td>
<td>(2.13)</td>
<td>(2.13)</td>
<td>(2.84)</td>
<td>(2.78)</td>
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<tr>
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<td>3.17</td>
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<tr>
<td>(1.52)</td>
<td>(1.49)</td>
<td>(2.79)</td>
<td>(3.62)</td>
<td>(3.11)</td>
<td>(4.68)</td>
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<td>log volume</td>
<td>1.27</td>
<td>0.29</td>
<td>3.68</td>
<td>5.84</td>
<td>5.17</td>
<td>7.56</td>
</tr>
<tr>
<td>(2.97)</td>
<td>(1.47)</td>
<td>(2.26)</td>
<td>(2.17)</td>
<td>(2.34)</td>
<td>(3.55)</td>
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<td>Stock liquidity</td>
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<tr>
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<td>12.94</td>
<td>10.92</td>
<td>12.43</td>
<td>10.91</td>
<td>12.37</td>
</tr>
<tr>
<td>(2.91)</td>
<td>(4.33)</td>
<td>(2.66)</td>
<td>(4.27)</td>
<td>(4.17)</td>
<td>(3.08)</td>
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<td>1.07</td>
<td>1.91</td>
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<tr>
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<td>(2.68)</td>
<td>(1.64)</td>
<td>(1.63)</td>
<td>(1.10)</td>
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### Table 7: Quarterly OTC volume change w.r.t to previous year

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<th>Q3</th>
<th>Q4</th>
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<tr>
<td>yty Change</td>
<td>44%</td>
<td>-60%</td>
<td>-40%</td>
<td>-23%</td>
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</table>
Table 8: Causal Impact of the Italian FTT - Derivatives

This table presents the estimates for the coefficient $\beta$ from specification (R.1), where the dependent variable corresponds to proxies for volume and liquidity described in section 8.1. The regression is applied to the introduction of the tax on derivatives written on Italian stocks and the event date is therefore 01.09.2013. Standard errors (in parenthesis) are clustered by stock and time and as usual ***, **, * denote statistical significance at the 1%, 5% and 10% level.

<table>
<thead>
<tr>
<th></th>
<th>Stocks</th>
<th>Options</th>
<th>Futures</th>
<th>OTC</th>
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<td><strong>Trading activity</strong></td>
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</tr>
<tr>
<td>log volume</td>
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<td>-0.211</td>
<td>-0.272</td>
<td>-0.0157</td>
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<tr>
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<td>(0.007)</td>
<td>(0.297)</td>
<td>(0.434)</td>
<td>(0.216)</td>
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<tr>
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<td></td>
<td></td>
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</tr>
<tr>
<td>log depth</td>
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<td>-0.075</td>
<td>0.246</td>
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</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>0.183</td>
<td>(0.129)</td>
<td></td>
</tr>
<tr>
<td>quoted spread</td>
<td>0.0361*</td>
<td>0.001</td>
<td>-0.003**</td>
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<td></td>
<td>(0.203)</td>
<td>0.0074</td>
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</table>
Figure 17: Graphical illustration of the FTT impact on French stock market measures
Figure 18: Graphical illustration of the FTT impact on French option market measures
Figure 19: Graphical illustration of the FTT impact on French futures market measures